STICKINESS

1. But Don't We *Know* Prices are Sticky?

Transactions prices, measured directly, might be far from the true spot prices of theory, and thus display a lot of stickiness whose real effect is small. We look at a model, discussed also in the first part of Sims (1998).

Consumer's objective:

$$\max_{C_s, L_s, B_s, Y_s} E\left[\sum_{t=0}^{\infty} \beta^t U(C_t, 1-L_t)\right]$$

Consumer's constraints:

$$\lambda: \qquad C_t + \frac{B_t}{P_t} + \tau_t \le \frac{Y_t}{P_t} + \pi_t + \frac{R_{t-1}B_{t-1}}{P_t}$$

$$\mu: \qquad Y_t \le w_t(L_t - (1-\delta)L_{t-1}) + (1-\delta)Y_{t-1}$$

Unusual variables: Y: wage bill; w: wage on new contracts; δ: rate of contract dissolution; τ: taxes.

FOC's:

$$\partial C: \qquad D_1 U_t = \lambda_t$$

$$\partial L: \qquad D_2 U_t = \mu_t w_t - \beta(1-\delta)E_t[\mu_{t+1} w_{t+1}]$$

$$\partial Y: \qquad \frac{\lambda_t}{P_t} = \mu_t - \beta(1-\delta)E_t \mu_{t+1}$$

$$\partial B: \qquad \frac{\lambda_t}{P_t} = \beta E_t\left[R_t \frac{\lambda_{t+1}}{P_{t+1}}\right]$$

Wage=MUL/MUC:

$$\frac{D_2 U_t}{D_1 U_t} = \frac{\dfrac{w_t}{P_t} - \beta(1-\delta)E_t\left[\dfrac{\mu_{t+1}P_{t+1}}{\mu_t P_t}\dfrac{w_{t+1}}{P_{t+1}}\right]}{1-\beta(1-\delta)E_t\left[\dfrac{\mu_{t+1}}{\mu_t}\right]} \tag{1}$$

Forward-looking:

$$
\begin{aligned}
w_t &= E_t \left[\sum_{s=0}^{\infty} \beta^s (1-\delta)^s \frac{D_2 U_{t+s} \mu_{t+s}}{\mu_t} \right] \\
\mu_t &= E_t \left[\sum_{s=0}^{\infty} \beta^s (1-\delta)^s \frac{D_1 U_{t+s}}{P_{t+s}} \right]
\end{aligned} \tag{2}
$$

3. The Firm

objective:

$$
\max_{L_s, Y_s, x_s} E \left[\sum_{t=0}^{\infty} \beta^t \Phi_t x_t \right]
$$

constraints:

$$
\begin{aligned}
\zeta: &\qquad x_t \leq A_t f(L_t) - \frac{Y_t}{P_t} \\
\nu: &\qquad Y_t \geq w_t (L_t - (1-\delta) L_{t-1}) + (1-\delta) Y_{t-1}
\end{aligned}
$$

FOC's:

$$
\begin{aligned}
\partial x: &\qquad \Phi_t = \zeta_t \\
\partial Y: &\qquad \frac{\zeta_t}{P_t} = \nu_t - \beta(1-\delta) E_t \nu_{t+1} \\
\partial L: &\qquad \zeta_t A_t f'(L_t) = \\
&\qquad \nu_t w_t - \beta(1-\delta) E_t [\nu_{t+1} w_{t+1}]
\end{aligned}
$$

Wage=MPL:

$$
A_t f'(L_t) = \frac{\dfrac{w_t}{P_t} - \beta(1-\delta) E_t \left[\dfrac{\nu_{t+1} P_{t+1}}{\nu_t P_t} \dfrac{w_{t+1}}{P_{t+1}} \right]}{1 - \beta(1-\delta) E_t \left[\dfrac{\nu_{t+1}}{\nu_t} \right]} \tag{3}
$$

Forward looking:

$$
\begin{aligned}
\nu_t = E_t \left[\sum_{s=0}^{\infty} \beta^s (1-\delta)^s \frac{\zeta_{t+s}}{P_{t+s}} \right] \\
= E_t \left[\sum_{s=0}^{\infty} \beta^s (1-\delta)^s \frac{D_1 U_{t+s}}{P_{t+s}} \right]
\end{aligned} \tag{4}
$$

4. Government

Budget Constraint:

$$\frac{B_t}{P_t} + \tau_t = R_{t-1}\frac{B_{t-1}}{P_t}$$

behavior: The results we are interested in do not depend on the details of government behavior, so long as it follows some fiscal policy that determines a unique price level — for example setting R and τ to be constants.

5. Interpretation

- Note that the right-hand sides of (3) and (1) are the same, except for the appearance of μ in the consumer version and ν in the firm version.
- The right-hand sides of (2) and (4) are also the same, so ν and μ are indeed the same. Here we use the usual trick of taking the firm's Φ_t to be equal to the consumer's $\lambda_t = D_1 U_t$.
- So we arrive at the usual equality between the marginal product of labor and the marginal value of leisure. This, together with the social resource constraint $C_t = A_t f(L_t)$ (which follows from the consumer constraint and the government budget constraint), delivers the usual efficient allocation, independent of the time path of prices.
- In this model, with a representative "household" that owns the firm, the only "real effect" of inflation is to redistribute wealth between household and firm. This has no effect on welfare because the household owns the firm. Labor contracts behave something like bonds: their value can be affected by surprise inflation.
- In a more general model, with incomplete insurance and asset markets, there would be real effects, but they would not be efficiency losses from $MPL \neq MUL/MUC$. They would be redistributional effects across agents holding different kinds of assets, where a labor contract is an asset.
- This model is not meant as realistic as it stands. It is only an example to show that observing sticky transactions prices (in wage contracts or catalogs, e.g.) does not prove that Keynesian stickiness is essential to understanding business cycles.
- A critical distinction: between contracts like those in this model that specify price *and quantity* and contracts that give the holder a quantity-unbounded "call option".

References

Sims, C. A. (1998): "Stickiness," *Carnegie-rochester Conference Series On Public Policy*, 49(1), 317–356.