"MICRO-FOUNDED" STICKINESS

1. DIXIT-STIGLITZ MONOPOLISTIC COMPETITION

Consumers' utilities depend on a consumption good aggregate

$$C_t = \left(\int_0^1 c_{jt}^{1-1/\theta} dj \right)^{\theta/(\theta-1)} ,$$

where j indexes a continuum of types of gooods and $\theta > 1$ determines how substitutable the goods are for one another. If good j has price j, consumers will maximize aggregate C subject to $\int p_j c_j \, dj = X$, where X is total expenditure on consumption goods. This implies that

$$C^{1/\theta} c_j^{-1/\theta} = \lambda p_j \,, \text{ all } j \,, \tag{$*$}$$

$$\int p_j c_j \, dj = C^{1/\theta} \cdot C^{\theta/(\theta-1)} / \lambda = C/\lambda = X \,. \tag{$**$}$$

2. PRICE INDEX

It is natural to think of the aggregate consumption price index as X/C, which makes it just λ in the above expression. It is not hard to show, using $(*)$ and $(**)$ that

$$X = PC = CP^{-\theta} \int p_j^{1-\theta} dj$$

$$\therefore P = \left(\int p_j^{1-\theta} dj \right)^{1/(1-\theta)}$$

3. FIRMS

Everyone hires labor in the same market at the same wage. (Different, and simpler than, Woodford's "base" model.)

With no frictions:

$$\max_{p_j} \left\{ p_j q_j - w\alpha q_j = (p_j - w\alpha) C \left(\frac{p_j}{P} \right)^{-\theta} \right\} ,$$

where q_j is quantity sold, w is the wage, and α is unit labor requirements. The solution is

$$p_j = \frac{\theta}{\theta - 1} w\alpha \,.$$

That is, price is a fixed markup over labor costs. Usually the "markup" is defined as $(p_j - w\alpha)/p_j = 1/\theta$, i.e. the proportion of the price that is not absorbed in variable costs.

4. Price adjustment friction

We have no nominal rigidity or money illusion yet. The Dixit-Stiglitz setup has just given us agents who set prices in an optimization problem. Possible devices:

- Prices can only be revised at fixed intervals. (Taylor, Stan Fischer, first case in Woodford book, "Tayor contracting")
- Firms can only revise prices at exogenously determined random times. ("Calvo pricing")
- Firms face costs of adjusting prices, with the costs increasing as the rate of change of price increases. ("Menu costs", quadratic adjustment costs)
- Firms face costs of adjusting prices, but choose the timing of price adjustments by comparing costs of price change to benefits of it. ("State-dependent pricing")
- Firms can only revise prices at fixed intervals, or pay a cost to increase the frequency of price revisions, but when they do revise, they set an optimal time path of prices, not a fixed price. (Mankiw-Reis inattention)
- Firms and/or consumers only loosely keep track of the determinants of optimal prices and quantities, because of (Shannon information theory based) costs of responding rapidly and accurately to information. (Sims "rational inattention")

5. Calvo pricing

A fixed, randomly selected, fraction δ of firms choose new prices each period. Since there are no costs associated with the size of the price change and all firms have the same demand and cost parameters, all the firms, no matter what their previous price, will choose the same current price p_t^*. The price dynamics will therefore be

$$\bar{P}_t = \delta p_t^* + (1-\delta)\bar{P}_{t-1} \qquad \text{or, in continuous time}$$

$$\dot{\bar{P}}_t = \delta(p^* - \bar{P}_t)\,. \tag{$*$}$$

Note that the $\bar{P}_t$ appearing here is the arithmetic average of prices whereas the exact price aggregate is a different average, closer to a harmonic mean. The formulas above are approximately correct for the exact price aggregate when price differences across firms are small.

6. Price setter's problem

A firm choosing its price at t maximizes with respect to p_{jt}

$$E_t\left[\int_0^\infty e^{-(\beta+\delta)t}\left(\frac{p_{jt}q_{j,t+s} - \alpha w_{t+s}q_{j,t+s}}{P_{t+s}}\right)ds\right]\,.$$

Note that we have them maximizing real (aggregate price deflated) profits. They discount by the consumers' discount rate β — which in a general equilibrium with risk averse agents is justifiable only as an approximation when uncertainty is not too great — plus the rate at which commitment to this fixed price is expected to decay in the future. Future quantities sold will determined be determined as $C_{t+s}(p_{jt}/P_{t+s})^{-\theta}$, that is by future aggregate consumption and future aggregate prices relative to the fixed p_{jt}.

7. p^*

$$p_t^* = \frac{\int_0^\infty \exp(-(\beta+\delta)s)\theta\alpha w_{t+s}P_{t+s}^{\theta-1}C_{t+s}\,ds}{\int_0^\infty \exp(-(\beta+\delta)s)(\theta-1)P_{t+s}^{\theta-1}C_{t+s}\,ds}$$

Log Linearization:

$$\tilde{p}_t^* = (\beta+\delta)\int_0^\infty e^{-(\beta+\delta)s}\tilde{w}_{t+s}\,ds$$

Here $\tilde{p}$ and $\tilde{w}$ are log deviations from steady state. (The version of these notes used in class mistakenly omitted the $\beta+\delta$ factor on the right-hand side.)

Substituting this in $(*)$ gives us a (much simplified) version of the New Keynesian Phillips curve:

$$\dot{\tilde{p}}_t = \delta\big((\beta+\delta)\int_0^\infty e^{-(\beta+\delta)s}\tilde{w}_{t+s}ds - \tilde{p}_t\big)$$

This equation makes $\tilde{p}$ adjust in the direction of the of the discounted present value of future wages. Note that the markup factor $\theta/(\theta-1)$ and the labor requirements factor α have dropped out of the equation because we are writing it in terms of log deviations from steady state. In more general models, with increasing costs rather than linear technology, it would be expected future marginal costs, rather than just wages, that would enter into the discounted present value. Expected future levels of output would affect future marginal costs. The standard Phillips curve therefore contains both expected future output and expected future wages.