EXERCISE ON PRICE DETERMINACY AND LINEAR RE SYSTEMS

Here is a New Keynesian model with fiscal policy and a government budget constraint introduced explicitly.

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\begin{aligned}
\text{Phillips Curve}: &\qquad \pi_t = \beta E_t \pi_{t+1} + \gamma y_t + \varepsilon_t &(1)\\
\text{Taylor Rule}: &\qquad r_t = \alpha \pi_t + \psi y_t + \delta r_{t-1} + \nu_t &(2)\\
\text{IS}: &\qquad r_t - E_t \pi_{t+1} = \bar{\rho} + \phi(E_t y_{t+1} - y_t] + \zeta_t &(3)\\
\text{GBC}: &\qquad b_t = \frac{1 + r_{t-1}}{1 + \pi_t} b_{t-1} - \tau_t &(4)\\
\text{Fiscal Policy}: &\qquad \tau_t = \theta_0 + \theta_1 b_t + \xi_t &(5)
\end{aligned}
$$

As a starting point, set these values for parameters:

$$\alpha = .15,\ \beta = .9,\ \gamma = .3,\ \psi = .1,\ \delta = .9,\ \bar{\rho} = \beta^{-1} - 1,\ \phi = .5,\ \theta_0 = -1,,\ \theta_1 = .2\,.$$

Assume that the disturbances ε, ν, ζ, and ξ are serially independent and mean zero.

(i) Explain how this model's Taylor rule satisfies the "Taylor principle".

(ii) The Taylor rule and the fiscal policy equation jointly fall into the "active money, passive fiscal" region as defined by Leeper. Explain.

(iii) Find the model's steady state and derive the linearization of the model around the steady state. Note that the model is already linear, except for the government budget constraint.

(iv) Translate this model into the notation of `gensys` and check whether it satisfies conditions for existence and uniqueness.

(v) Now set $\alpha = .05$. Show that the Taylor principle is no longer satisfied. Check what `gensys` says about existence and uniqueness.

(vi) Keeping $\alpha = .05$, set $\theta_0 = 1$, $\theta_1 = 0$. Show that we are now in the "active fiscal, passive money" region. Check what gensys says about existence and uniqueness.

(vii) Now set $\alpha = .15$ again. Check what `gensys` says about existence and uniqueness.

(viii) Returning all parameters to their original values, explore the effects of changing α alone. Is the boundary at which indeterminacy appears exactly the "Taylor principle" boundary, or have the New Keynesian complications made that boundary only approximate?

(ix) Some analyses suggest that the best-performing monetary policies involve making the coefficient on lagged interest rates in the Taylor rule (here δ) greater than one. What happens in this model with $\delta > 1$?

(x) What are the impact effects and the first one or two subsequent effects of changes in ν_t and ξ_t? Do these match conventional beliefs about the effects of "tightening money" or "loosening fiscal policy"?

Hints: When The equations with expectational terms have been time-shifted to match `gensys` notation, their exogenous shock terms will be lagged. You can handle this two ways. One is to introduce two new variables and dummy equations: e.g. ν_t^*, with dummy equation $\nu_t^* = \nu_{t-1}$. This makes the system bigger, but makes interpretation of the output possibly easier. The other way is to leave the system as it is, but recognize that the exogenous shock terms that enter the two time-shifted equations are known one period in advance. Then one uses the `ywt,fmat,fwt` part of the solution to find the time-t effects of ν_t and ζ_t. In the approach that adds dummy equations, all the time-t effects of shocks are read off of the `impact` matrix returned by `gensys`.

It is probably easiest to eliminate τ from the system by substituting the fiscal policy equation into the government budget constraint.