ECO 503 - Fall 2007
Assignment 2 - Professor C. Sims

The system of equations describing the model is:

$$\begin{aligned}
\pi_t &= \beta E_{t\pi t+1} + \gamma y_t + \varepsilon_t & (1)\\
r_t - \overline{\rho} &= \alpha \pi_t + \psi y_t + \delta \left(r_{t-1} - \overline{\rho} \right) + v_t & (2)\\
r_t - E_t \pi_{t+1} &= \overline{\rho} + \varphi (E_t y_{t+1} - y_t) + \zeta_t & (3)\\
b_t &= \frac{1 + r_{t-1}}{1 + \pi_t} b_{t-1} - \tau_t & (4)\\
\tau_t &= \theta_0 + \theta_1 b_t + \xi_t & (5)
\end{aligned}$$

1. With the initial values of the parameters the long run response of the nominal interest rate to inflation is equal to $1.5 = \frac{\alpha}{1-\delta} = \frac{0.15}{0.1}$. Hence the Taylor principle is satisfied given that the nominal interest rate responds more than one-for-one to an increase in inflation.

2. From question 1 we know that the Taylor principle is satisfied. Leeper's criterion for passive fiscal policy is also satisfied:

$$|\beta^{-1} - \theta_1| = |10/9 - .2| = 0.9111 < 1$$

Therefore, monetary and fiscal rules fall into the "active monetary, passive fiscal" region.

3. Steady states:

$$\begin{aligned}
\overline{\pi} &= 0\\
\overline{y} &= 0\\
\overline{r} &= \overline{\rho}\\
\overline{b} &= \frac{\theta_0}{\overline{\rho} - \theta_1}\\
\overline{\tau} &= \frac{\overline{\rho}\theta_0}{\overline{\rho} - \theta_1}
\end{aligned}$$

Substituting (5) in (4) and linearizing around the steady state we get:

$$\begin{aligned}
\widetilde{\pi}_t &= \beta E_t \widetilde{\pi}_{t+1} + \gamma \widetilde{y}_t + \varepsilon_t\\
\widetilde{r}_t &= \alpha \widetilde{\pi}_t + \psi \widetilde{y}_t + \delta \widetilde{r}_{t-1} + v_t\\
\widetilde{r}_t - E_t \widetilde{\pi}_{t+1} &= \varphi (E_t \widetilde{y}_{t+1} - \widetilde{y}_t) + \zeta_t\\
\widetilde{b}_t &= \frac{\overline{b}}{1+\theta_1} \widetilde{r}_{t-1} - \overline{b} \frac{1+\overline{\rho}}{1+\theta_1} \widetilde{\pi}_t + \frac{1+\overline{\rho}}{1+\theta_1} \widetilde{b}_{t-1} - \xi_t
\end{aligned}$$

4. Define the state vector:

$$S_t = \left[\widetilde{\pi}_t, \widetilde{y}_t, \widetilde{r}_t, \widetilde{b}_t, E_t\widetilde{\pi}_{t+1}, E_t\widetilde{y}_{t+1}\right]$$

and the vector containing the exogenous shocks:

$$e_t = [\varepsilon_t, \upsilon_t, \zeta_t, \xi_t]$$

The system of equations can be written in canonical form:

$$\Gamma_0 S_t = \Gamma_1 S_{t-1} + C + \Psi e_t + \Pi \eta_t$$

$$\Gamma_1 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & cb_r & cb_b & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad \Psi = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad \Pi = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$\Gamma_0 = \begin{bmatrix} 1 & -\gamma & 0 & 0 & -\beta & 0 \\ -\alpha & -\psi & 1 & 0 & 0 & 0 \\ 0 & \phi & 1 & 0 & -1 & -\phi \\ -cb_\pi & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \eta_t = \begin{bmatrix} \widetilde{\pi}_t - E_{t-1}\widetilde{\pi}_t \\ \widetilde{y}_t - E_{t-1}\widetilde{y}_t \end{bmatrix}$$

$$cb_\pi = \bar{b}\frac{1+\bar{\rho}}{1+\theta_1}, \; cb_r = \frac{\bar{b}}{1+\theta_1}, \; cb_b = \frac{1+\bar{\rho}}{1+\theta_1}$$

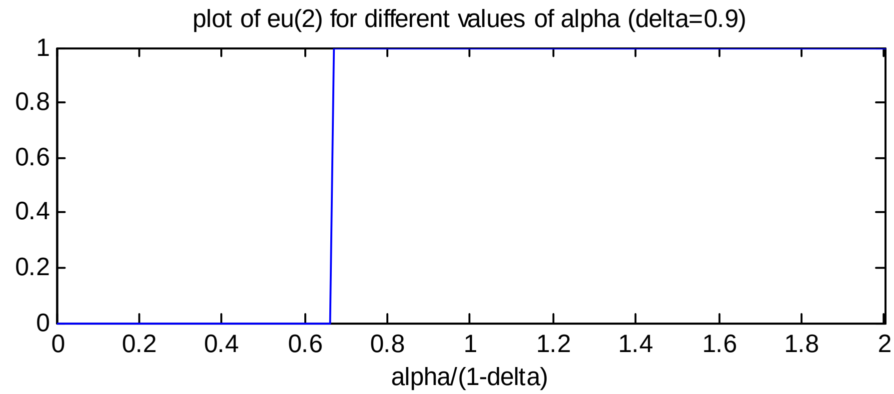
Under the current parameters ($\alpha = 0.15$, $\theta_0 = -1$, $\theta_1 = .2$) gensys returns $eu = (1,1)$: existence and uniqueness. Note that this is consistent with the results of Leeper given that we are in the "active monetary, passive fiscal" region.

5. With $\alpha = 0.05$, $\theta_0 = -1$, $\theta_1 = .2$ we have that both monetary policy and fiscal policy are passive. Accordingly gensys returns $eu = (1,0)$: indeterminacy (the solution is not unique).

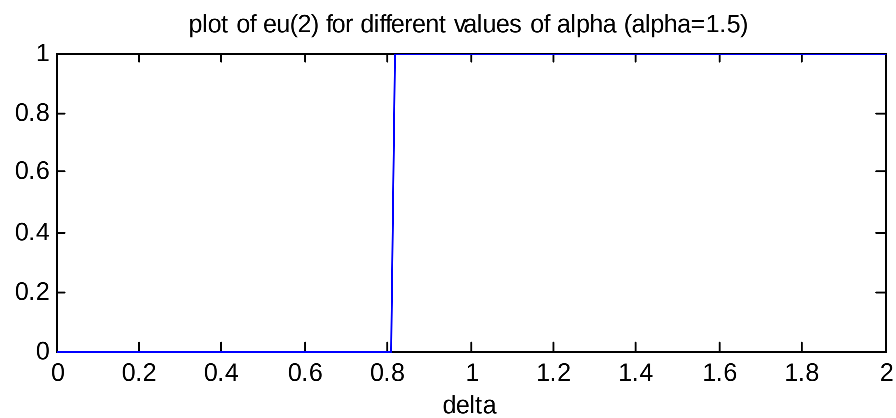
6. With $\alpha = 0.05$, $\theta_0 = 1$, $\theta_0 = 0$ we have that monetary policy is passive while fiscal policy is active ($|\beta^{-1} - 0| = 10/9 > 1$). Gensys returns $eu = (1,1)$: existence and uniqueness.

7. With $\alpha = 0.15$, $\theta_0 = 1$, $\theta_0 = 0$ we have that both monetary policy and fiscal policy are active. Gensys returns $eu = (0,1)$: no existence. Note how all previous results are in line with the findings of Leeper (1991).

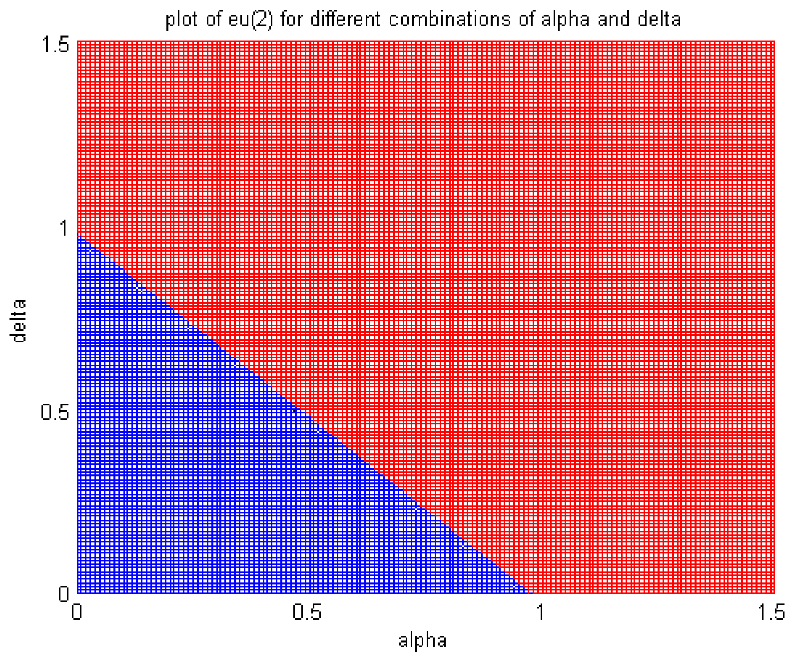
8. The following graph plots eu(2) for different values of α. The horizontal axis reports $\alpha/(1-\delta)$. The Taylor principle threshold is $\overline{\alpha}=0.1$, i.e. $\overline{\alpha}/(1-\delta)=1$. The "Taylor principle" holds only approximately given that the threshold for determinacy turns out to be around $\alpha=0.067$, i.e. $\overline{\alpha}/(1-\delta)=0.67$.

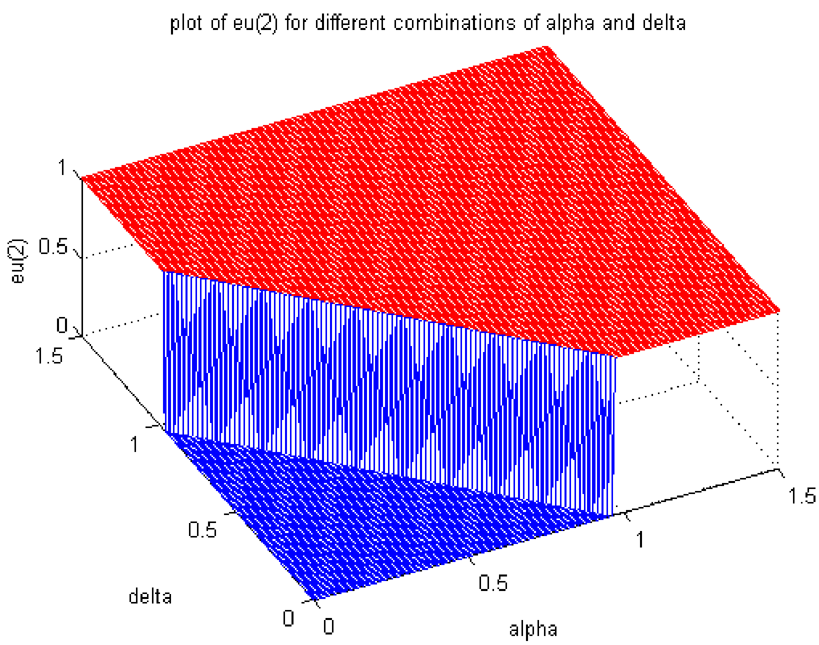


9. The graph below reports $eu(2)$ for different values of delta once α has been set equal to 0.15. The solution is unique for any value of δ larger than .82 (this is an approximation based on the grid used in the simulation). Note that $eu(1)=1$ for each $\delta>0$: a solution always exists.

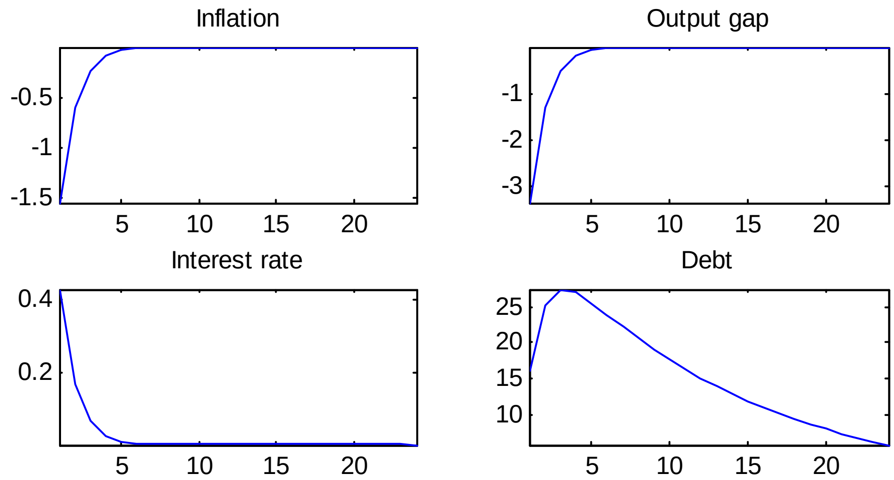


The following graphs report the value of eu(2) for different combinations of α and δ. The red (light) area corresponds to eu(2)=1 (determinacy), while the blue (dark) area corresponds to eu(2)=0 (indeterminacy). Note that with $\delta>1$, we get determinacy and uniqueness for all values of $\alpha>0$. A $\delta>1$ implies that after an inflationary shock the monetary authority keeps rising the nominal interest rate for several periods.


plot of eu(2) for different combinations of alpha and delta
1.5
1
0.5
0
delta
0
0.5
1
1.5
alpha

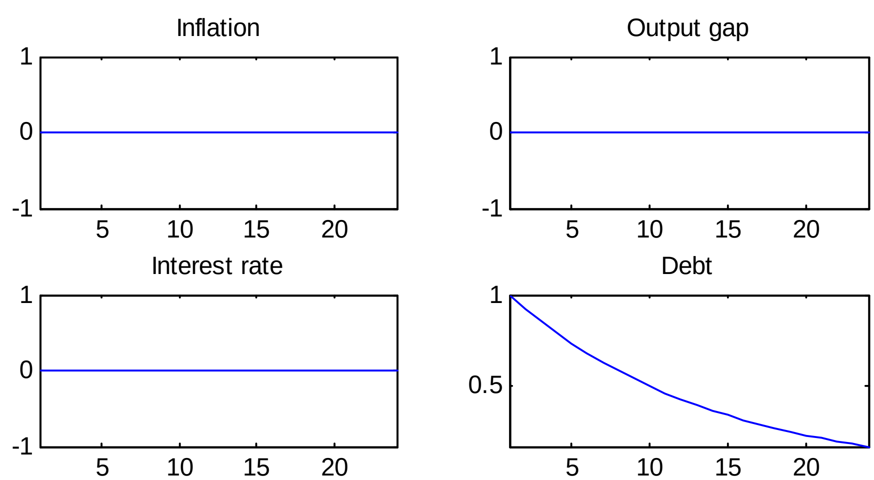

plot of eu(2) for different combinations of alpha and delta
1
0.5
0
eu(2)
1.5
1
0.5
0
delta
0
0.5
1
1.5
alpha

10. The following graphs report the impulse responses of the four endogenous variables to a monetary policy shock. A tightening monetary policy determines a decrease in inflation and a contraction in output. Moreover, debt increases on impact and it keeps rising for a while due to the increase in interest payments. These results match conventional beliefs about the effects of a monetary policy shock. In the long run all variables go back to their steady state values.



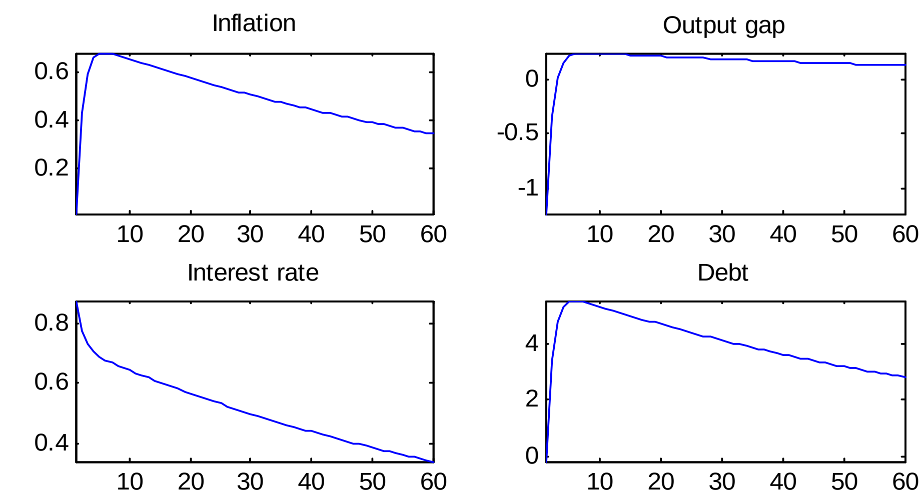
Monetary policy shock (Active monetary, passive fiscal)

The following graphs report the impulse responses of the four endogenous variables to a fiscal policy shock. An expansionary fiscal policy determines an increase in debt but it has no effect on current output and inflation. This contradicts conventional beliefs. However, it is in line with the Ricardian theory of debt. In the long run all variables go back to their steady state values.



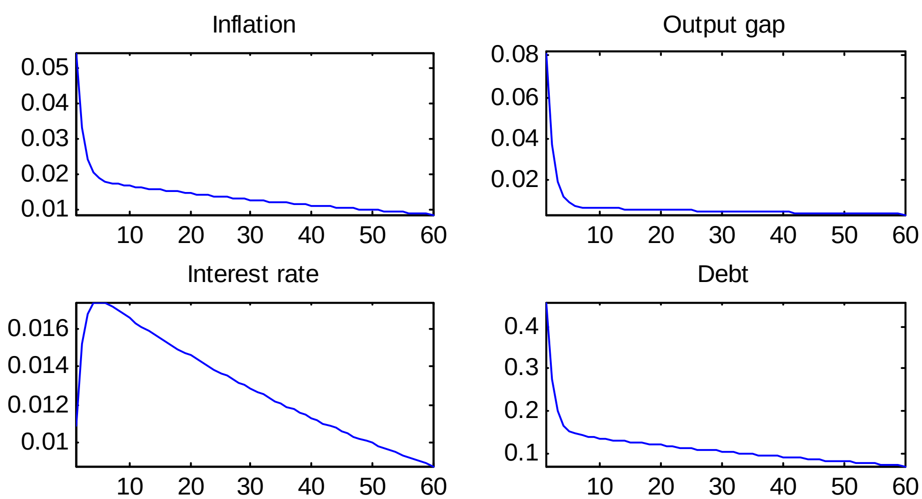
Fiscal policy shock (Active monetary, passive fiscal)

Let's now consider the "active fiscal, passive monetary" case. The following graphs report the impulse responses of the four endogenous variables to a monetary policy shock. A tightening monetary policy determines an increase in debt and a contraction of output. The increase in inflation is not in line with common beliefs about the effects of monetary policy but it is consistent with the Fiscal Theory of Price Level. In the "active fiscal, passive monetary" case, an increase in the interest rate determines an increase in debt without any real contractionary effect.



Monetary policy shock (Active fiscal, passive monetary)

The following graphs report the impulse responses of the four endogenous variables to a fiscal policy shock. Following an expansionary fiscal policy inflation and output go up. This determines an increase in the interest rate. These results match conventional beliefs about the effects of an expansionary fiscal policy. In the long run all variables go back to their steady state values.



Fiscal policy shock (Active fiscal, passive monetary)