Fiscal and Monetary Interactions

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January 17, 2008

1 The Taylor Principle for Taylor Rules

Why it works: a simple model

$$
\begin{aligned}
&\max_{C,B,M} E\left[\sum_{t=0}^{\infty}\beta^t \log C_t\right] \text{ s.t.} \\
C_t(1+\gamma v_t) + \frac{B_t+M_t}{P_t} &= \frac{R_{t-1}B_{t-1}+M_{t-1}}{P_t} + Y_t + g_t \\
v_t &= \frac{P_tC_t}{M_t} \\
R_t &= \beta^{-1}\left(\frac{P_tC_t}{P_{t-1}C_{t-1}}\right)^{\theta} && \text{Taylor Rule} \\
\frac{B_t+M_t}{P_t} &= \frac{R_{t-1}B_{t-1}}{P_t} + g_t && \text{Gov't Budget Constraint} \\
g_t &= g_0 - \phi\frac{B_{t-1}}{P_{t-1}} + \varepsilon_t && \text{Fiscal Policy}
\end{aligned}
$$

FOC's are:

$$\partial C: \qquad \frac{1}{C_t} = \lambda_t(1+2\gamma v_t) \tag{1}$$

$$\partial B: \qquad \frac{\lambda_t}{P_t} = \beta R_t E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] \tag{2}$$

$$\partial M : \qquad \frac{\lambda_t(1-\gamma v_t^2)}{P_t} = \beta E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] . \tag{3}$$

From the FOC's and the policy rule we can produce

$$R_t = (1-\gamma v_t^2)^{-1} \tag{4}$$

$$\left(\frac{1-\gamma v_t^2}{\beta}\right)^{1-1/\theta} Z_t = E_t Z_{t+1} \tag{5}$$

$$Z_t = \frac{(1-\gamma v_t^2)^{1/\theta}}{1+2\gamma v_t} . \tag{6}$$

Here (4) is a "liquidity preference" or "demand for money" function, derived from the B and M FOC's. Equation (6) just defines notation. Equation (5) comes from the M FOC, using the policy rule to substitute out $P_tC_t/(P_{t+1}C_{t+1})$ and (4) to substitute out R. NB: Z_t monotone decreasing in v_t. (5) has a solution with constant Z (and hence constant v) and is locally unstable if $\theta > 1$ (the Taylor principle). We treat Y_t and ε_t as i.i.d.

But can we rule out the locally unstable paths as equilibrium solutions?

- The paths in which Z increases, and hence v decreases, can be ruled out. Once Z goes above its steady state, equilibrium requires that it be unbounded above, but this cannot happen even with $v \to 0$.
- The paths in which Z decreases, and hence v increases, cannot be ruled out. On these paths, v approaches a finite upper limit as $Z \to 0$, while R and P_t/P_{t-1} approach infinity. No feasibility constraint is violated if such a path persists forever, with ever accelerating inflation.

Same model, pure interest rate peg

- The stationary equilibrium has R, M, PC and PY constant.
- We have not used the government budget constraint or the fiscal rule. They simply determine a stationary time path for government debt.
- What if policy were not a Taylor-principle Taylor rule, but instead $R_t = \beta^{-1}$, i.e. a pure interest rate peg?

- What if, further, fiscal policy were to make the primary deficit (in equilibrium a surplus, if debt is positive) exogenous, but following the same stochastic process (as a function of Y_t and ε_t) as in the Taylor-principle Taylor rule?

- Answer: Equilibrium is *exactly the same*.

Uniqueness

- The Taylor-principle Taylor rule equilibrium price level is not unique.

- This interest-rate peg equilibrium does deliver a unique price level.

- The unstable equation is no longer the Z equation, but the government budget constraint.

- Deflationary deviations in which real debt explodes upward are ruled out by transversality.

- Inflationary deviations in which real debt shrinks toward zero are ruled out as infeasible from the viewpoint of private agents — they would see themselves as having insufficient resources, in real bonds and discounted present value of Y, to support both the SRC level of C and the discounted value of current and future taxes g_t.

- So they would reduce their demand, reduce prices, bring the price level back to the equilibrium path.

Fixed M

If monetary policy is to fix M rather than to apply an interest rate rule, results are different. We can for this case again derive a single-variable nonlinear difference equation that lets us analyze existence and uniqueness. In place of (6) we use

$$Z_t = \frac{1}{v_t(1 + 2\gamma v_t)} \tag{7}$$

and, by simply multiplying the M FOC by the constant M, arrive at

$$(1 - \gamma v_t^2) Z_t = \beta E_t Z_{t+1} \,. \tag{8}$$

This is again a locally unstable difference equation with a unique steady state at $1 - \gamma v_t^2 = \beta$. As before, solutions with $Z_t \to \infty$ and (therefore) $v_t \to 0$ can be ruled out. However, while in the Taylor-rule case they could be ruled out by the fact that the Z for that case is by construction bounded above, here one has to invoke a transversality argument. Z can go to infinity, but that requires real balances to go to infinity, and no path with bounded C and unbounded real balances can be optimal. With large enough M/P, the gains in current utility from turning some of the real balances into current consumption must dominate the discounted present value of future increased transactions costs due to the lower real balances. Z going to zero can now be ruled out as infeasible, however. With this monetary policy, $Z \to 0$ requires $v \to \infty$, but (8) cannot be satisfied with $v_t > 1/\sqrt{\gamma}$. The behavioral interpretation of this is that there is a minimal level of real balances in this model that people are willing to hold even if they know that money will next period become worthless. With fixed M, this puts an upper bound on P in any equilibrium.

Both the fixed-M and Taylor-Principle-Taylor-rule policies are "active" in Leeper's sense — they guarantee a locally unique stable equilibrium. But these two active policy rules, in a fully fleshed out equilibrium model, have different implications for whether unstable paths can be ruled out.

Kocherlakota-Phelan uniqueness

The non-uniqueness with Taylor-rule monetary policy can be resolved by fiscal policy. With passive fiscal policy, the non-uniqueness will be there. But with active fiscal policy, there may be a unique, though possibly explosive, equilibrium. For example, suppose that fiscal policy fixes at σ the primary surplus plus the real value of seignorage, evaluated at the shadow value of consumption λ_t. In other words, it sets

$$\lambda_t \tau_t = \frac{\tau_t}{C_t(1+2\gamma v_t)} \equiv \sigma \, .$$

This lets us multiply the government budget through by λ_t and rewrite it as

$$\frac{\lambda_t B_t}{P_t} = R_{t-1} \frac{\lambda_{t-1} B_{t-1}}{P_{t-1}} \frac{\lambda_t P_{t-1}}{\lambda_{t-1} P_t} - \sigma \, . \tag{9}$$

Taking E_{t-1} of this equation and using the bond FOC, we get

$$E_{t-1} \frac{\lambda_t B_t}{P_t} = \beta^{-1} \frac{\lambda_{t-1} B_{t-1}}{P_{t-1}} - \sigma \tag{10}$$

This equation has a unique stable solution for initial $\lambda_t b_t = \lambda_t B_t / P_t$, namely $\lambda_t b_t \equiv \sigma\beta/(1-\beta)$. We can rule out $\lambda_t b_t$ going to infinity at the rate β^{-t} by a transversality argument: the conditions for the standard TVC apply, and the standard TVC here is $E[\beta^t \lambda_t b_t] \to 0$. We can rule out its going to minus infinity by the constraint $B_t > 0$. So b_t is uniquely determined and constant. Substituting this determinate value into the left-hand-side of (9), we have an equation that involves lagged data, constants, and $\lambda_t / P_t = 1/(P_t C_t (1 + 2\gamma v_t))$. But $P_t C_t / (P_{t-1} C_{t-1})$ determines, via the Taylor rule, R_t, which in turn, via the liquidity preference relation (4), determines v_t, and these relations are all monotonic. Thus everything on the right-hand-side can be expressed in terms of v_t, so v_t is uniquely determined, as are R_t and $P_t C_t$. But $P_t C_t = P_t Y / (1 + \gamma v_t)$. Thus given $P_t C_t$, v_t, and the exogenously fixed Y, we can solve for P_t, and the P_t time path is uniquely determined.

If the initial v_t determined this way implies a Z_t below the steady-state value of Z_t (here using the Taylor-rule, not the fixed-M, definition of Z), then we are on a unique, explosive, equilibrium time path for the price level. [Note: It is uniquely determined, but depends on the lagged values $P_{t-1}C_{t-1}$ and b_{t-1}]. But if the implied initial Z_t is above the steady-state Z_t, there is no equilibrium (i.e. we have non-existence), because a high initial Z_t implies positive probability of arbitrarily large Z_t according to the monetary part of the model, and Z_t is by construction bounded above.

Kocherlakota and Phelan, in the paper we discussed in class, have a model (with a simpler demand for money) that generates a unique equilibrium like this — unique, but explosive, despite an active monetary policy. They argue that this equilibrium is implausible, because in their case it arises despite a fixed money stock. It seems to them somehow unbelievable that the real money stock could shrink to zero, and that while that happens the price level blows up. But in their model, as in this one, the equilibrium with explosive inflation involves explosively growing conventional (not primary) deficits. Real revenues plus seignorage exceed expenditure and are stable, but interest expense explodes and nominal debt (because real debt is constant as the inflation rate explodes upward) explodes. To observers of the economy, it would (rightly) appear that exploding fiscal deficits are driving the inflation despite stable (in this model) or shrinking (in Kocherlakota-Phelan) real balances.

Identification

- The fact that the same observed behavior of the variables can emerge either from active-money/passive-fiscal policy combinations or the reverse is not a special result of a trick model. It is generally true that a given equilibrium generated by active monetary and passive fiscal policy can be supported also by active fiscal, passive monetary policy combinations.

- This does not rule out identification. We may know something about differences in what fiscal and monetary authorities care about, or what they observe, or the nature of delays in their decision-making, that allows identification, or at least allows exploring hypothetical identifications.