EXERCISE: RISK SHARING

In class we discussed a simple model with two countries (or two types of agents) in which each country i's representative agent solves

$$\max C_{it}, B_{it} \sum_{t=0}^{\infty} \beta^t U(C_{it}) \quad \text{subject to} \tag{1}$$

$$C_{it} + B_{it} = R_{t-1} B_{i,t-1} + Y_{it} \tag{2}$$

and the traded risk-free asset B is in zero net supply. The lecture characterized the solution of the linearized model for this case, without giving a full derivation. In the linearized model, with Y_t i.i.d. with support included in $\mathbb{R}^+$, C and B are both martingales, implying they do not converge and therefore must eventually violate any boundedness conditions on them that are present in the original nonlinear version of the model.

(i) Assume $U() = \log()$. Solve the model linearized around $B = 0$ and verify the conclusions in the lecture notes. A solution will specify C_{it} and B_{it} as functions of lagged data and current exogenous shocks (here Y_{it} itself).

We noted in the lecture that in the original nonlinear model, neither agent can in fact issue truly risk-free debt, so the linearized model has to be thought of as approximating a model in which there is default risk, but only at levels of B much higher than the initial $B = 0$ level about which we linearize.

(ii) Consider the same model, but now with the exogenous Y_{it} process following $Y_{it} = Y_{i,t-1}^{\theta} \varepsilon_{it}$, where ε_{it} is i.i.d. with $E_{t-1}\varepsilon_{it} = 1$. Solve the linearized model and show that now the linearized model shows no borrowing or lending in equilibrium. Why does this change in the exogenous process for Y make such a difference?

Now consider a model in which, instead of an endowment that arrives exogenously, there is a technology in each country that uses capital to produce the consumption good, but that still only the risk-free bond is traded. Specifically, the country i budget constraint is now

$$C_{it} + K_{it} + B_{it} = A_{it} K_{i,t-1}^{\alpha} + R_{t-1} B_{i,t-1} \,, \tag{3}$$

where $\alpha \in (0, 1)$.

(iii) Solve the linearized model for the case where A_{it} is i.i.d. with mean 1. Continue to assume B is in zero net supply. Linearize around $B_{it} = 0$. Is there borrowing in this linearized model?

(iv) Repeat the analyis in (iii), but now assuming $A_{it} = A_{i,t-1}^{\theta} \varepsilon_{it}$, where ε_{it} is i.i.d. with $E_{t-1}\varepsilon_{it} = 0$ and $\theta \in (0, 1)$.

(v) In the setup of (iii) and (iv), is there the same problem as in the classroom problem that individuals cannot in fact issue truly risk-free debt securities? Why or why not.

In the linearized models, you can if you like assume numerical values for the parameters and use `gensys` to find the solution. Use $\alpha = .3$, $\beta = .95$, $\theta = .9$, and a non-stochastic steady-state Y_{it} equal to 1.