

**RANDOM LAGRANGE MULTIPLIER AND TRANSVERSALITY
EXERCISE**

(1) The standard analytically solvable stochastic growth model is

$$\max_{\{K_t, C_t\}} E \left[\sum_{t=0}^{\infty} \beta^t \log C_t \right] \quad \text{subject to}$$

$$C_t + K_t = A_t K_{t-1}^\alpha, \quad t = 0, \dots, \infty$$

K_{-1} and the distribution of $\{A_t\}$ given.

- (a) Find the Euler equation first order conditions for a solution.
- (b) Verify that there is a solution to the Euler equations and the constraint that makes C_t proportional to K_t at every t .
- (c) Verify that the conditions for the application of the standard TVC are present and that it is satisfied at the solution you have found with C proportional to K . Do you have to invoke regularity conditions on the stochastic process followed by A_t in order to get your result? If so, what are they?
- (d) (*Extra credit in case you are aching to apply your rusty real analysis tools.*) Define a linear space whose points map one to one into pairs of $\{C_t\}$ and $\{K_t\}$ sequences and a metric on the space such that the objective function in this problem is continuous and the constraint set is contained entirely in the linear space. Verify the concavity of the objective function and the convexity of the constraint set. Find the continuous linear functional that separates the constraint set from the set of points preferred to the optimum. Discuss how this functional is related to the Euler equations and the TVC.