CAPITAL TAX EXERCISE

We consider a simple frictionless growth model with capital but no labor. The government, for reasons we leave unspecified, has to finance a given stochastic time path of transfer payments g_t by use of the only tax available to it, a proportional tax on capital set aside this period for use next period.

The representative private consumer-producer solves

$$\max_{c_t,b_t,k_t} E\left[\sum_{t=0}^{\infty} \beta^t \log c_t\right] \qquad \text{subject to} \tag{1}$$

$$c_t + (1+\tau_t)k_t + b_t = a_t k_{t-1}^{\alpha} + \delta k_{t-1} + r_{t-1}b_{t-1} + g_t\,. \tag{2}$$

The government is benevolent, so it also maximizes (1), but it chooses, in addition to the private sector's choice variables, the tax rate τ_t, and the (real) interest rate r_t. It does not choose g_t — it takes that as given. The government's constraints are all the private sector constraints and FOC's, plus the government budget constraint

$$b_t + \tau_t k_t = g_t + r_{t-1}b_{t-1}\,. \tag{3}$$

We assume the exogenous shocks a_t and g_t are i.i.d., with means 1 and 0, respectively.

(a) Find the Euler equation first order conditions (FOC's)for the consumer.

(b) Find the Euler equation FOC's for the government.

(c) Verify that the problem has a deterministic steady state at $\tau = b = 0$ and solve for the steady state values of c, k, and r for this case. [Hint: Find formulas for steady state r and k as a function of the parameters, then express steady state c as a function of steady state k. Lagrange multipliers on private FOC's should emerge as zero in this steady state.]

(d) Show that the system of FOC's also has a steady state at every other constant positive value of τ. In this case, it is not necessary to solve for the steady state explicitly. Just show that the equations of the system can be solved for this case. Note that for this to be true, at least one of the equations defining steady state must be redundant.

(e) Assume $\alpha = .3$, $\beta = .95$, $\delta = .93$. Linearize the system around the steady state you found in (c) and use `gensys` or another linear rational expectations solver to find the impact effects of the two exogenous shocks (`impact` in the `gensys` output) and $G_1^s x$, $s = 1, \ldots, 3$, where x is the impact matrix from `gensys` and G_1 is `G1` from gensys. Together, what you have calculated will be the first four periods' responses of all variables to the exogenous shocks. Observe whether the optimal time path of the capital tax is in fact decreasing in this linearized solution.

(f) The result that the optimizing government might leave τ constant at a non-zero level seems to clash with Chamley's classic result. Can you explain which aspects of this model are central to the differing conclusion?

Note: I am only 95% sure that the things you are asked to prove in this exercise are true.