FTPL EXERCISE

In the following model, we generalize the model discussed in class by allowing an arbitrary utility function, while simplifying by eliminating money. We consider a combination of "active money" with "passive fiscal" policies, in Leeper's terminology.

The agent maximizes

$$E\left[\sum_{t=0}^{\infty}\beta^t U(C_t)\right] \tag{1}$$

subject to

$$C_t + \frac{B_t}{P_t} + \tau_t = \frac{R_{t-1}B_{t-1}}{P_t} + Y_t \tag{2}$$

$$B_t \geq 0, \text{ all t } . \tag{3}$$

The government budget constraint is

$$\frac{B_t}{P_t} + \tau_t = \frac{R_{t-1}B_{t-1}}{P_t} . \tag{4}$$

Suppose the government's monetary and fiscal policies are to set

$$R_t = \left(\frac{U'(C_t)}{P_t}\right)^{-\theta} \tag{5}$$

$$\tau_t = -\phi_0 + \phi_1 \frac{B_{t-1}}{P_{t-1}} \tag{6}$$

Assume $\theta > 0$, so R increases when P increases, and $\phi_0 > 0$, $\phi_1 > \beta^{-1} - 1$.

(a) Show that there is only one equilibrium in which prices, taxes and debt follow stable paths. [Hint: Log the Fisher equation and use Jensen's inequality: $\log(E[X]) \geq E[\log(X)]$ because log() is concave. Use the result to derive relations of the form $E[X_{t+1}] >= X_t$ for both the positive and negative parts of $\log(U'(C_t)/P_t)$. Then use the fact that if $E_t[X_{t+1}] \geq X_t$ for all t and X_t is bounded above for all t, then X_t converges a.s.]

(b) Show that equilibrium is nonetheless not unique.

Comment: Note that in the model of the lectures uniqueness depended sensitively on money being "essential". Here money is very much non-essential, being non-existent.