EXERCISE ON RANDOM LAGRANGE MULTIPLIERS AND TVC

(1) Consider the following variant of the standard LQ permanent income model, in which we use a different form of the accumulation constraint from that used in class, and we relax the condition $R\beta = 1$:

$$\max_{\{C_t\}} E_0 \left[\sum_{t=0}^{\infty} \beta^t (C_t - \tfrac{1}{2} C_t^2) \right] \tag{1}$$

$$\text{subject to } C_t + A_t \leq R A_{t-1} + Y_t \tag{2}$$

$$A_t \geq 0 \tag{3}$$

$$Y_t > 0 \text{ with probability one }, \ EY_t < \infty,\ Y_t \ i.i.d. \tag{4}$$

(a) Show that the objective function in this modified model is concave.
(b) Find the Euler equations and transversality conditions.
(c) Find the optimal decision rule, setting C_t as a function of A_{t-1} and Y_t, for the "standard" form of the model, in which we replace (3) by $E[\beta^{t/2} A_t] \to 0$ and make (1) an equality, instead of an inequality.
(d) Show that your solution to the standard problem does not solve the problem in this exercise.

(2) Consider the simple "new Keynesian" model

$$\text{aggregate demand}: \quad y_t = \beta E_t y_{t+1} - \theta(r_t - E_t \pi_{t+1}) + \nu_t \tag{5}$$

$$\text{Phillips curve}: \quad \pi_t = \delta E_t \pi_{t+1} + \gamma y_t + \varepsilon_t \tag{6}$$

$$\text{Taylor rule}: \quad r_t = \alpha_1 \pi_t + \alpha_2 y_t + \alpha_3 r_{t-1} + \zeta_t\ . \tag{7}$$

There are no constant terms because all variables are interpreted as deviations from a steady state. Use a computer — `gensys.m` will work fine — to complete the following tasks.

(a) Check existence and uniqueness for the model with $\beta = .95$, $\theta = .5$, $\delta = .8$, $\gamma = .2$; $\alpha_1 = .11$, $\alpha_2 = .01$, $\alpha_3 = .9$.
(b) For these same parameter values, compute and plot impulse responses of r, π, and y to the three shocks ε, ν, ζ, which are all interpreted as i.i.d.
(c) Determine what range of parameter values for α_1 and α_2 are consistent with existence and uniqueness. Does the "Taylor Principle", that $\alpha_1/(1-\alpha_3)$ should exceed 1, provide a necessary and sufficient condition?

Note that, because ε and ν enter with a t subscript earlier than the date on the latest variables to appear in their equations, if you use `gensys` they have to be treated as variables in the system, appearing with a lag, and dummy equations have to be added to the system that set them equal to i.i.d. shocks.

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