1. Transversality conditions, "no-Ponzi" conditions, and "intertemporal budget constraints"

Borrowing Constraints: When agents are modeled as able to raise resources by issuing securities or borrowing, there must be some constraint that prevents their raising arbitrarily large resources by issuing arbitrarily large amounts of securities. Otherwise they will consume, or issue dividends (in the case of firms) in arbitrarily large, and infeasible, amounts. Our examples have used simple deterministic bounds on borrowing: $B_t \geq -\bar{B}$ or $C_t \leq W_t$.

2. Intertemporal budget constraints

- Relate the borrowing constraint to ability to pay.
- A complete-markets notion.
- Firm: market net worth remains non-negative
- Consumer: with a market stochastic discount factor Φ_t,

$$B_t \leq E_t \sum_{s=1}^{\infty} \beta^s \frac{\Phi_{t+s}}{\Phi_t} \left(Y_{t+s} - C_{t+s}\right) . \tag{1}$$

  for any feasible consumption plan. That is, this constraint defines the set of feasible consumption plans.
- Corresponds to a period budget constraint and no-Ponzi condition of the form

$$C_t \leq Y_t - \beta^{-1} \Phi_t^{-1} B_{t-1} + B_t \tag{2}$$

$$\lim_{T \to \infty} E_t \left[\beta^T \Phi_{t+T} B_{t+T} \right] \leq 0 . \tag{3}$$

- Verifiable at t because there are markets at t in which the Φ_t values corresponding to every future contingency are quoted and because all probabilities are known. If (as usual in such a model) $C_t < 0$ is impossible but C_t arbitrarily close to 0 is possible, then we just check, by looking at the market value of traded assets, that

$$B_t \leq E_t \sum_{s=1}^{\infty} \beta^s \frac{\Phi_{t+s}}{\Phi_t} Y_{t+s}$$

3. No-Ponzi conditions

- Hybrids of the ITBC and $B \leq \bar{B}$.
- Apply to incomplete-markets situations.
- No uniquely accurate or reasonable way to set them up.

- With single-period budget constraint $B_t = C_t - Y_t + R_{t-1}B_{t-1}$, solve forward to obtain

$$B_t = \sum_{s=1}^{T} \Big(\prod_{v=1}^{s} R_{t+s-1}^{-1} \Big) (Y_{t+s} - C_{t+s}) + \Big(\prod_{s=1}^{T} R_{t+s-1}^{-1} \Big) B_{T+1} \,. \quad (4)$$

- If the last term in this expression goes to zero as $T \to \infty$, get something like (1).
- Equivalent to what is usually called the no-Ponzi condition, which is the requirement that the last term in (4) go to zero as $T \to \infty$.
- Important differences from (1):
  - the condition is often written (as here) with no expectation operator in front of it;
    - With an E in front, it seems to imply that running a risk that something will happen that makes payment impossible is OK, so long as that is offset by a sufficient probability of having more than enough resources to pay. This implicitly assumes that B is *not* a risk-free bond.
    - Without an E in front, it is extremely restrictive, often implying an agent can't take out any loans at a risk-free rate, because it requires that even in extremely improbable worst-case scenarios the agent must be able to pay back the debt for sure.
  - the discounting is done using some existing market return (here R_t), not the ideal complete-markets stochastic discount factor.

4. Something completely different

Transversality. Borrowing constraints, no-Ponzi conditions, and intertemporal budget constraints are all inequalities, bounding debt or net worth from below. They are perceived by agents as set externally. The TVC is a condition for optimization, in most economic models playing the role of ruling out solutions in which wealth grows rapidly forever but is never used to provide consumption or dividends.

5. Real Business Cycles

What is the real business cycle theory or school?

- It might seem obvious: an approach that attempts to explain business fluctuations as efficient responses of producers and consumers to random variation in the technological environment. And this characterization is partially correct.
- But there are papers that are by RBC economists and in the RBC style that explore sticky prices (Chari, Kehoe and McGrattan, e.g.) and that explore the implications of financial frictions (several papers by Christiano and Eichenbaum, e.g.). So what else characterizes the RBC style?
  - stochastic general equilibrium modeling ;

- much more readiness to devote resources to computation of solutions to nonlinear GE models, and to simplifying models so that such solutions are possible;
- willingness to leave prices out of the model, particularly the overall price level, and to ignore implications of the model for price behavior;
- adherence to "calibration" rather than "estimation and testing" as the criterion for assessing a model's fit;

All these criteria have become fuzzier over time, with some of the RBC characteristics becoming common outside the school (like stochastic GE modeling and, sadly, calibration) and some outside characteristics (like price stickiness and statistical assessment of fit) showing up at least occasionally in RBC work.

6. But Don't We *Know* Prices are Sticky?

Transactions prices, measured directly, might be far from the true spot prices of theory, and thus display a lot of stickiness whose real effect is small.

Consumer's objective:

$$\max_{C_s,L_s,B_s,Y_s} E\left[\sum_{t=0}^{\infty} \beta^t U(C_t, 1-L_t)\right]$$

Consumer's constraints:

$$\lambda: \qquad C_t + \frac{B_t}{P_t} + \tau_t \le \frac{Y_t}{P_t} + \pi_t + \frac{R_{t-1}B_{t-1}}{P_t}$$

$$\mu: \qquad Y_t \le w_t(L_t - (1-\delta)L_{t-1}) + (1-\delta)Y_{t-1}$$

Unusual variables: Y: wage bill; w: wage on new contracts; δ: rate of contract dissolution; τ: taxes.

7.

FOC's:

$$\partial C: \qquad D_1 U_t = \lambda_t$$

$$\partial L: \qquad D_2 U_t = \mu_t w_t - \beta(1-\delta)E_t[\mu_{t+1} w_{t+1}]$$

$$\partial Y: \qquad \frac{\lambda_t}{P_t} = \mu_t - \beta(1-\delta)E_t \mu_{t+1}$$

$$\partial B: \qquad \frac{\lambda_t}{P_t} = \beta E_t\left[R_t \frac{\lambda_{t+1}}{P_{t+1}}\right]$$

Wage=MUL/MUC:

$$\frac{D_2U_t}{D_1U_t} = \frac{\dfrac{w_t}{P_t} - \beta(1-\delta)E_t\left[\dfrac{\mu_{t+1}P_{t+1}}{\mu_t P_t}\dfrac{w_{t+1}}{P_{t+1}}\right]}{1 - \beta(1-\delta)E_t\left[\dfrac{\mu_{t+1}}{\mu_t}\right]} \tag{5}$$

Forward-looking:

$$\begin{aligned} w_t &= E_t\left[\sum_{s=0}^{\infty}\beta^s(1-\delta)^s\frac{D_2U_{t+s}}{\mu_t}\right] \\ \mu_t &= E_t\left[\sum_{s=0}^{\infty}\beta^s(1-\delta)^s\frac{D_1U_{t+s}}{P_{t+s}}\right] \end{aligned} \tag{6}$$

8. The Firm

objective:

$$\max_{L_s,Y_s,x_s} E\left[\sum_{t=0}^{\infty}\beta^t\Phi_t x_t\right]$$

constraints:

$$\begin{aligned} &\zeta: && x_t \le A_t f(L_t) - \frac{Y_t}{P_t} \\ &\nu: && Y_t \ge w_t(L_t - (1-\delta)L_{t-1}) + (1-\delta)Y_{t-1} \end{aligned}$$

FOC's:

$$\begin{aligned} &\partial x: && \Phi_t = \zeta_t \\ &\partial Y: && \frac{\zeta_t}{P_t} = \nu_t - \beta(1-\delta)E_t\nu_{t+1} \\ &\partial L: && \zeta_t A_t f'(L_t) = \\ & && \quad \nu_t w_t - \beta(1-\delta)E_t[\nu_{t+1}w_{t+1}] \end{aligned}$$

Wage=MPL:

$$A_t f'(L_t) = \frac{\dfrac{w_t}{P_t} - \beta(1-\delta)E_t\left[\dfrac{\nu_{t+1}P_{t+1}}{\nu_t P_t}\dfrac{w_{t+1}}{P_{t+1}}\right]}{1 - \beta(1-\delta)E_t\left[\dfrac{\nu_{t+1}}{\nu_t}\right]} \tag{7}$$

Forward looking:

$$\nu_t = E_t\left[\sum_{s=0}^{\infty}\beta^s(1-\delta)^s\frac{\zeta_{t+s}}{P_{t+s}}\right]$$
$$= E_t\left[\sum_{s=0}^{\infty}\beta^s(1-\delta)^s\frac{D_1U_{t+s}}{P_{t+s}}\right] \quad (8)$$

9. Government

Budget Constraint:

$$\frac{B_t}{P_t} + \tau_t = R_{t-1}\frac{B_{t-1}}{P_t}$$

behavior: The results we are interested in do not depend on the details of government behavior, so long as it follows some fiscal policy that determines a unique price level — for example setting R and τ to be constants.

10. Interpretation

- Note that the right-hand sides of (7) and (5) are the same, except for the appearance of μ in the consumer version and ν in the firm version.
- The right-hand sides of (6) and (8) are also the same, so ν and μ are indeed the same. Here we use the usual trick of taking the firm's Φ_t to be equal to the consumer's $\lambda_t = D_1U_t$.
- So we arrive at the usual equality between the marginal product of labor and the marginal value of leisure. This, together with the social resource constraint $C_t = A_t f(L_t)$ (which follows from the consumer constraint and the government budget constraint), delivers the usual efficient allocation, independent of the time path of prices.
- In this model, with a representative "household" that owns the firm, the only "real effect" of inflation is to redistribute wealth between household and firm. This has no effect on welfare because the household owns the firm. Labor contracts behave something like bonds: their value can be affected by surprise inflation.
- In a more general model, with incomplete insurance and asset markets, there would be real effects, but they would not be efficiency losses from $MPL \neq MUL/MUC$. They would be redistributional effects across agents holding different kinds of assets, where a labor contract is an asset.
- This model is not meant as realistic as it stands. It is only an example to show that observing sticky transactions prices (in wage contracts or catalogs, e.g.) does not prove that Keynesian stickiness is essential to understanding business cycles.
- A critical distinction: between contracts like those in this model that specify price *and quantity* and contracts that give the holder a quantity-unbounded "call option".