EXERCISE ON EXPECTATIONS AND ADVICE

This is a practice exercise related to the last lecture. It need not be handed it. It is a little more complicated than a likely question on a 3-hour final, but it may help you understand the issues raised in the last lecture.

Note that the questions on the last three years' final exams for this course are also good sources of practice exercises, though OG questions are overrepresented on those exams relative to the content of this year's course.

We consider the overworked natural rate Phillips curve model:

$$u_t = \bar{u} - \theta(\pi_t - E_{t-1}\pi_t) \,. \tag{1}$$

The central bank has the loss function

$$E\left[\sum_{t=0}^{\infty} \beta^t (u_t^2 + \pi_t^2)\right] \,, \tag{2}$$

which of course it wants to minimize.

(a) Suppose the central bank always has either a $\pi_t = 1$ policy or a $\pi_t = 0$ policy. The conditional probability of sticking with the same policy next period, given policy this period, is always .9, so policy choices are persistent. With this specification it can be shown that $P[\pi_{t+s} = 1 \mid \pi_t = 1] = .5(1 + .8^s)$. Using symmetry and the fact that probabilities sum to one, we can derive the following table showing the probabilities of various pairs π_{t+s}, π_{t+s+1} conditional on the two possible values of π_t.

π_t	π_{t+s}	π_{t+s+1}	prob
1	1	1	$.45(1+.8^s)$
1	1	0	$.05(1+.8^s)$
1	0	0	$.45(1-.8^s)$
1	0	1	$.05(1-.8^s)$
0	0	0	$.45(1+.8^s)$
0	0	1	$.05(1+.8^s)$
0	1	1	$.45(1-.8^s)$
0	1	1	$.05(1-.8^s)$

A member of the bank's board has the opportunity at t to cast the deciding vote to determine π_t. She has no illusions that this will change the stochastic process governing future choics of π_t. Both she and the public understand that the stochastic process will remain the same after her vote. Nonetheless her vote will affect the conditional expectation of the loss function. Calculate the expected loss conditional on each value of π_t as a function of π_{t-1} and θ.

(b) Continuing under the assumptions of (a), determine whether there is any value of θ such that if this board member were choosing π every period with this loss function, the actual stochastic process for π would match that specified in (a).
(c) Still continuing with the (a) assumptions, suppose that there is a stochastic political process by which our decision maker with loss function (2) alternates her possession of the deciding vote with another decision maker with a different loss function. Can you specify a loss function for the other decision maker and a stochastic process for the power switches that would make the process specified in (a) the truth?
(d) Suppose that the decision maker in (a) assumes that the public always expects at time t that the π_{t-1} level of inflation will persist, and she herself believes that her deciding vote will permanently determine the level of inflation from t onwwards. How does the mapping from π_{t-1} and θ to current action change?
(e) Compare your analysis here to the conclusions that emerge from a "no-commitment" or "full-commitment" assumption as in the lecture notes on policy games.