EXERCISE: CONSUMPTION SMOOTHING

We discussed the nature of a solution to the simple linearized international borrowing and lending model we considered in class, but didn't actually lay out a solution. In this problem, you will find the actual solution. Note that the 2002 version of the course included a similar, but not identical exercise, for which an answer was posted on the course web page. This year's version involves a little more algebra, but should be easier if you start from the old solution.

The model has agent i, $i = 1, 2$ solving

$$\max_{C_i, B_i} E\left[\sum_{t=0}^{\infty} \beta^t \frac{C_i(t)^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

subject to

$$C_i(t) + B_i(t) = R_{t-1} B_i(t-1) + Y_i(t) \tag{2}$$

$$B_i(t) \geq -\bar{B}\,. \tag{3}$$

We assume that the bonds are privately issued, so that $B_1(t) = -B_2(t)$ is the market clearing condition. Assume that Y_1 and Y_2 are independent of each other, both evolving according to the same stochatic process, with mean $\bar{Y} > 0$ and satisfying

$$Y_i(t) = \rho(Y_i(t-1) - \bar{Y}) + \bar{Y} + \varepsilon_i(t)\,,$$

where $\varepsilon_i(t)$ is i.i.d. across both i and t and has mean zero.

(i) Linearize the model around a deterministic steady state in which $B = 0$, and solve for C_1, C_2, R, and B_1 as functions of the history of the exogenous processes Y_i.

(ii) Check whether in your linearized solution B and $C_1 - C_2$ are martingales and R is i.i.d., as was true in the 2002 $\rho = 0$ version of the exercise.

(iii) In lecture it was asserted that the solution gets closer to autarchy as ρ increases toward one. Check whether that is true in your solution and explain how you reach your conclusion.

Note that this is a model in which there is a deterministic steady state at each value of B, so that the linearized solution will drift away from the initial steady state.

This problem can be solved straightforwardly with `gensys.m`*, but for this exercise you are to show how to do it by hand. If you want to try it without looking at the 2002 solution to a similar problem, you should still simplify notation by replacing the two B_i variables by a single B that enters the two constraints with opposite signs, thereby allowing you to drop the market clearing equation. Also, define $\tilde{C}(t) = .5(C_1(t) - C_2(t))$ and $\hat{C}(t) = .5(C_1(t) + C_2(t))$ and transform the model so these variables replace C_1 and C_2.*