

CAPITAL TAXATION EXERCISE

Consider a representative agent economy in which the consumer's budget constraint is

$$C_t + K_t = (1 - \tau_t)K_{t-1}^\alpha + g_t. \quad (1)$$

τ_t is a tax rate on capital in use at t and g_t is a transfer payment from the government. We assume the government budget constraint is

$$\tau_t K_{t-1}^\alpha = g_t, \quad (2)$$

i.e. that every period the government turns over all its tax revenue as lump sum transfers to the consumers. Note that the individual consumers nonetheless assume that as they vary K_t , they change their time-($t+1$) tax obligations without changing their own g_{t+1} value.

Suppose the consumer maximizes

$$V = \sum_{t=0}^{\infty} \beta^t \log(C_t). \quad (3)$$

- (a) Show that if τ_t is constant, equilibrium will require K_t/C_t is constant at, say $K_t/C_t = \kappa$. Find κ as a function of the model's parameters. Assume $0 < \alpha < 1$ and $0 < \beta < 1$.
- (b) Show that C_t converges to a constant long-run value \bar{C} and find \bar{C} as a function of model parameters.
- (c) Show that $d\bar{C}/d\tau < 0$
- (d) Is it true in this model, as was claimed in class, that $dV(\tau, K_{-1})/d\tau = 0$, if we evaluate the derivative at $\tau = 0$? Is this true regardless of the initial value of K ?
- (e) Suppose that the government must make a transfer payment in the amount g_0 at time zero, but that it can issue debt, so it is not necessary that tax revenue cover the transfer payment at time zero. There will be no transfers after time 0 ($g_t = 0$ for $t > 0$.) The only tax available is the capital tax. Negative capital taxes are not possible. What is the optimal time path for the capital tax, assuming perfect foresight on the part of the public?
- (f) Suppose that there is an implementation delay for the capital tax, so that the government sees $\tau_0 = 0$ as a constraint. It therefore must finance the entire g_0 amount with debt issue, but then is free to tax in any time pattern it likes. What is the optimal time path of the capital tax in this case?

Note: For the last two parts of the problem where we introduce debt, the term $B_t - R_{t-1}B_{t-1}$ is added to the left-hand side of both the private agents' and the government's budget constraint. This means the capital tax does not apply to wealth in the form of government bonds. We assume initial debt is zero, so raising revenue in the first period by repudiating old debt is not possible. We also are assuming real debt, though this

makes no difference in this setting without uncertainty. A government without initial debt that can commit, as we assume here, will find that varying the price level has no effect on real outcomes, so that any time path for the price level is optimal.