CAPITAL TAXATION EXERCISE

Suppose the representative agent solves

$$\max_{C_t, K_t, B_t} \sum_{t=0}^{\infty} \beta^t \log C_t \quad \text{subject to} \tag{1}$$

$$C_t + B_t + K_t = (1 - \tau_t) r_t K_{t-1} + \pi_t + R_{t-1} B_{t-1} \,. \tag{2}$$

The firm solves the static problem

$$\max_{k_t} k_t^{\alpha} - r_t k_t = \pi_t \,. \tag{3}$$

The firm turns over its profits π_t to its owner, the representative agent. Market clearing requires that capital in use at t by the firm be the same as capital acquired last period and rented out this period by the consumer, i.e. $k_t = K_{t-1}$.

The government will pick a constant capital tax rate $\tau_t = \bar{\tau}$ subject to its budget constraint

$$B_t + \bar{\tau} r_t K_{t-1} = G_t + R_{t-1} B_{t-1} \,. \tag{4}$$

We assume that the government's spending obligations are proportional to output, so that it is an exogenous constraint that

$$G_t = \kappa K_{t-1}{}^{\alpha} \,. \tag{5}$$

(a) Show that in this model a fixed tax rate on capital will keep tax revenue a fixed proportion of output, so that if $B_{-1} = 0$ a fixed tax rate can be chosen that maintains budget balance forever, even if K varies over time.

(b) With $B_t \equiv 0$ and a fixed $\bar{\tau}$ set to balance the budget, calculate the value of the consumer's objective function (i.e. the discounted present value of $\log C$) as a function of the initial capital stock and κ.

(c) While maintaining the assumption that G must be kept proportional to output, calculate the value of the consumer's objective function, again as a function of initial capital stock and κ, for the case where the government has lump sum taxes available to use in place of the capital tax.

(d) Is it true in this model that the welfare losses from capital taxation are second-order in $\bar{\tau}$ for small τ? Note that we are not comparing here the utility with and without government spending, but rather comparing the utility at a given κ value between the lump-sum taxing and the capital-taxing versions of the model. The question is whether this difference shrinks to zero as $0(\bar{\tau}^2)$ as $\bar{\tau}$ (and hence κ) go to zero. Explain how you arrived at your answer.

(e) Suppose the government can own private assets, that it is not constrained to keep τ constant, and that it can announce any time path for τ it likes and be believed. However still the only way for it to tax (or transfer funds back to the public) is via the tax on capital (or its negative, a subsidy to capital). What is the optimal time path for τ?