LINEARIZING AND SOLVING AN RBC MODEL

In this exercise you will linearize and solve a dynamic stochastic equilibrium model, using the result to answer some questions about how certain "stylized facts" about cyclical fluctuations relate to the structural parameters of such a model.

1. The Model

We will use CES (constant elasticity of substitution) aggregators in several places. To hold down the notation, we will write these as

$$Y(x,z;a,b) = (ax^b + (1-a)z^b)^{1/b}\,.$$

Here $1-b$ is the **elasticity of substitution** between x and y when $b \le 1$, as it must be to make the mapping from z and x to Y concave. When $b \ge 1$, the mapping is convex, as is appropriate if Y is being interpreted as an aggregate output constraint giving substitution tradeoffs among component products. As $b \to 0$, $Y(x,z;a,b) \to x^a z^{1-a}$, i.e. to a Cobb Douglas production function.

At the left of each constraint we display the suggested Greek letter to use for the constraint's multiplier in forming the Lagrangian.

1.1. The Consumer.

$$\max_{C,L,S} E\left[\sum_{t=0}^{\infty} \beta^t \frac{Y(C_t, 1-L_t;\theta,\mu)^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

$$\tag{2}$$

subject to

$$\lambda: \qquad C_t + P_t S_t \le w_t L_t + (P_t + \delta_t) S_{t-1}\,. \tag{3}$$

There must also be some sort of borrowing constraint. $S_t \ge 0$, all t, will work, and because of the market clearing condition (below) we know it will not be binding in equilibrium. The substitution parameter μ characterizes how leisure and consumption combine to produce utility, so $\mu \le 1$ (making the indifference curves convex to the origin.)

1.2. The Firm.

$$\max_{K,L,x} E\left[\sum_{t=0}^{\infty} \beta^t \Phi_t x_t\right] \tag{4}$$

subject to

$$\zeta: \qquad Y(x_t + w_t L_t, I_t; \phi, \nu) \le A_t Y(K_{t-1}, L_t; \alpha, \sigma) \tag{5}$$

$$\xi: \qquad K_t \le I_t + \psi K_{t-1}\,. \tag{6}$$

We assume, as is often done, that $\Phi_t = \lambda_t$ from the consumer's problem, as would emerge from complete asset markets, even though we have no explicit model of the complete markets. A_t is an i.i.d. sequence of random technology shocks with mean 1. The substitution parameter ν is greater than or equal to 1, because it characterizes the transformation curve between investment goods and consumption goods. $1-\sigma$, on the other hand, is a production function elasticity of substitution, so $\sigma \le 1$.

1.3. **Market Clearing.**

$$x_t = \delta_t \tag{7}$$

$$S_t = 1\,. \tag{8}$$

The firms are owned by the consumers in the form of equity. The number of shares outstanding is 1.

1.4. **Your Task.** You will use a program that solves arbitrary linear rational expectations models that can be put into the form

$$\Gamma_0 y_t = \Gamma_1 y_{t-1} + \Psi \varepsilon_t + \Pi \eta_t\,, \tag{9}$$

where $E_t\varepsilon_{t+1} = E_t\eta_{t+1} = 0$. The difference between the two error terms is that ε_t is exogenously given, while η_t has to be determined as a function of ε_t in the solution. Usually, ε_t terms arise from the occurrence in the model of an expression of the form $E_{t-1}y_t$, which is then replaced by the equivalent $y_t - \eta_t$, where $\eta_t = y_t - E_{t-1}y_t$ is the one-step-ahead forecast error (or **innovation**) in y_t.

You are to derive Euler equations and transversality conditions that define equilibrium for this model, find the model's steady state, linearize the Euler equations about the steady state, and finally place the results in the form (9). The TVC's of course do not directly enter the linearized model. They just provide insight into what stability conditions can justifiably be imposed on the linearized model. Are the conditions for use of the simplified ŞstandardŤ TVC conditions satisfied?

You should start with the following parameter values:

α: .3
σ: -1.0
β: .95
γ: 2.0
θ: .7
μ: -1.0
ϕ: .7
ν: 2.0
ψ: .9

Using these parameter values, does the linearized model make the conventional $E_tC_{t+1} = C_t$ approximation for consumption behavior approximately correct? Is your conclusion on this score sensitive to the choice of γ, σ, μ or ν? The most direct way to check this is to calculate the variance of C_t and it first few autocovariances (covariance of C_t with C_{t-s}). If the martingale approximation is good, The autocovariances will be nearly the same as the variance. Also when you calculate the regression coefficient $\mathrm{Cov}(C_t, C_{t-s})/\mathrm{Var}(C_{t-s})$, it should be close to one for each s.

2. Practical Details

For the last part of the problem, you will need to use the program `gensys.m` (unless you are already familiar with one of the other available general linear rational equations system solvers). You will also need to use a program that calculates autocovariances from the output of `gensys`. This can be done with the Matlab routine `dlyap`, which however is in an add-on package, not the core Matlab program, and therefore may not be available to you. It can also be done with `lyapcsd`.

The theory underlying the `gensys` program will not be discussed until Tuesday or Thursday (4/9 and 4/11). However, most of the work in this exercise will probably be in getting the FOC's and the steady state calculation programmed. You should start working on this immediately, so that this part of the problem can be discussed in precept Tuesday.

Most of what this exercise asks you to do is already done in another, somewhat simpler, model in the Matlab file `jacobianex.m`, which is explained in the notes titled "Setting Up a Model for Analytic Linearization".