OLG EXERCISES

(1) Suppose in our overlapping generations model the utility function is

$$U\big(C_1(t), C_2(t+1)\big) = \log\big(C_1(t) \cdot C_2(t+1)\big)\,. \tag{A}$$

Suppose also that instead of being endowed with $\bar{Y}$ when young and nothing when old, each generation is endowed with $\bar{Y}/2$ when young and the same amount when old. Suppose the population is constant and $\theta > 1$.

(a) Show that in this case the policy of financing initial-period government spending g_0 entirely with debt issue and taxing only at some later date $T > 0$ will not work.

(b) Suppose that debt and taxes are constrained to be non-negative, but that the old as well as the young can be taxed. Find a sequence of taxes and debt issue that finance the initial g_0 by imposing the burden only on generation $T > 0$, without imposing any burden on generations before or after T.

(2) Consider an economy that begins at time $t = 0$ with an old generation that owns a fixed quantity R_0 per capita of a natural resource and a young generation that has a production technology. The natural resource can be stored without cost from one period to the next, or it can be used in the production technology to produce the consumption good. The old can sell the natural resource to the young. The problem for generation t is therefore

$$\max_{C_1(t), C_2(t+1), R_t, R_{t+1}} U\big(C_1(t), C_2(t+1)\big) \tag{B}$$

subject to

$$C_1(t) + P_t R_t = f(R_t - R_{t+1}) \tag{C}$$

$$C_2(t+1) = P_{t+1} R_{t+1} \tag{D}$$

$$R_t \geq R_{t+1} \geq 0\,. \tag{E}$$

(a) Assuming the functional forms $U(x, y) = \log(xy)$ and $f(r) = r^\alpha$, with $\alpha < 1$, find the competitive equilibrium time paths of R, P, C_1 and C_2, as well as of generational utilities.

(b) Suppose we are interested in finding a fiscal policy that will result in a less rapid decline in utilities across generation. Here are two proposed policies:

- A tax on the old, proportional to their holdings of R carried over from the first period of life, with the proceeds distributed as a lump sum transfer to the young;
- A tax on the young, proportional to their usage of natural resources (i.e. to $R_t - R_{t+1}$), with the proceeds distributed as a lump sum to the old.

Would either policy succeed in reducing the rate of decline of utility? Would either be Pareto-efficient?

(c) If the old as well as the young had access to the production technology, would the first generation simply use up all the resources? Why or why not?