CAPITAL TAXES

1. Why Capital Taxes Are Different

- Static analysis suggests that deadweight loss from taxation at rate τ is $0(\tau^2)$ — that is, that for small tax rates the ratio of deadweight loss to revenue is arbitrarily small.
- Consider a labor tax. Its effects on work, L, are $0(\tau)$. The equilibrium condition is $(1-\tau)f'(L) = -D_L U/D_C U$. But since the distortion involves trading C for leisure, and in the neighborhood of $\tau = 0$ the leisure gained is of the same utility as the consumption lost, the effects on welfare are small. There is of course a loss due to the C that gets appropriated for G, unless the government is optimizing so that G has utility that compensates for the decline in C. But the *additional* loss due to use of labor rather than lump-sum taxes is second-order.
- Consider the steady-state effects of a tax on capital income. The equilibrium condition is $(1-\tau)D_k f(K,L) = \beta^{-1} + \delta - 1$. ($\delta$ is the depreciation rate.) The tax decreases the steady-state capital-labor ratio and thereby also lowers the equilibrium wage and equilibrium L. The decline in C due to the decline in L is a second-order effect, but the decline that comes about because of the K/L decrease is not. In other words, steady-state C declines by more than can be compensated by the increase in steady-state leisure. This means steady-state U declines by more than is necessary to allow for the government spending, and the effect is first-order.
- But, a one-time, surprise capital levy is completely non-distorting.
- A temporary capital tax has the same $0(\tau^2)$ deadweight loss behavior as a labor tax.
- A permanent capital tax has the same $0(\tau^2)$ deadweight loss behavior as a labor tax, when this is measured in terms of discounted utility. The long run decline in utility from lower consumption in the future is, in the neighborhood of $\tau = 0$, exactly compensated for by the temporary rise in utility as dissaving allows temporarily higher consumption.

2. The nature of a optimal taxes

Capital tax τ:

- Optimal τ is zero in the long run.
- It is as high as you like right now.
- This raises problems of time consistency.
- There is no steady state with fixed optimal $\tau \neq 0$.
- Optimality of socialism?

Labor tax ψ:

- There is a steady state with $\tau \equiv 0$ and fixed $\psi \neq 0$ — one for each B.

Why the difference? The effects of "compounding".: A constant proportional capital tax changes the relative prices of current and future consumption. The effect is small in the current period, and over any finite number of future periods. But no matter how small τ is, $(1-\tau)^n$ eventually (for large enough n) is closer to zero than to one. So the distortion in the relative prices of present and future consumption is large for the distant future, even when τ is small. With discounted utility, this doesn't matter because the big distortions are also heavily discounted.

3. A MODEL IN WHICH WE SEE THAT OPTIMAL K TAXES DECLINE

The intuition: if the government has available some other source of finance that doesn't distort intertemporal choice (even if it is distortionary in a static sense), or if the government can shift revenue in time via issuance of debt or accumulation of assets, capital taxes should decline so as to leave intertemporal choices undistorted.

4. THE CONSUMER

$$\max_{C,L,B,K} E\left[\sum_{t=0}^{\infty} U(C_t,L_t)\beta^t\right] \tag{1}$$

$$C_t + B_t + K_t - \delta K_{t-1} \le (1-\tau_t)r_t K_{t-1} + (1-\psi_t)w_t L_t + R_{t-1}B_{t-1} \tag{2}$$

$$B_t \ge 0 \qquad K_t \ge 0\,. \tag{3}$$

FOC's after solving to eliminate Lagrange multipliers (and assuming the positivity constraints on assets (3) don't bind):

$$D_C U_t = \beta E_t\left[D_C U_{t+1}\big((1-\tau_{t+1})r_{t+1}+\delta\big)\right] \tag{4}$$

$$D_C U_t = \beta R_t E_t[D_C U_{t+1}] \tag{5}$$

$$\frac{D_L U_t}{D_C U_t} = -(1-\psi_t)w_t\,. \tag{6}$$

5. THE FIRM

$$\max_{M_t,L_t}\{f(M_t,L_t) - w_t L_t - r_t M_t\}\,, \tag{7}$$

where M_t is the number of "machines" rented this period by the firm. Market clearing requires that $M_t = K_{t-1}$, i.e., that the machines demanded by the firms as input match the supply of available machines that the consumer "saved up" last period. We assume that f is homogeneous of degree 1, so that we don't have to keep track of any dividends — profits are zero in equilibrium. The firm's problem generates, from solved FOC's,

$$w_t = D_L f(K_{t-1}, L_t) = D_L f_t \tag{8}$$

$$r_t = D_K f(K_{t-1}, L_t) = D_K f_t \,. \tag{9}$$

6. Government

The government shares the representative consumer's objective function (1), and it has the constraints

$$\mu: \qquad B_t + \tau_t r_t K_{t-1} + \psi_t w_t L_t \tag{10}$$

$$= B_t + \tau_t D_k f_t K_{t-1} + \psi_t D_L f_t L_t \geq R_{t-1} B_{t-1} + g_t$$

$$B_t \geq 0 \qquad K_t \geq 0 \,. \tag{11}$$

We suppose that the government takes spending requirements g_t as an exogenous stochastic process, not subject to choice. From the government budget constraint (10) and the consumer's constraint, we can derive the social resource constraint, which is a simpler replacement for the consumer's budget constraint in the government's optimization problem. From the private FOC's we can eliminate r and w to emerge with the remaining constraints on the government:

$$\lambda: \qquad C_t + K_t - \delta K_{t-1} + g_t \leq f(K_{t-1}, L_t) \,. \tag{12}$$

$$\nu: \qquad D_C U_t = \beta E_t \left[D_C U_{t+1} \big((1 - \tau_{t+1}) D_K f_{t+1} + \delta \big) \right] \tag{13}$$

$$\theta: \qquad D_C U_t = \beta R_t E_t [D_C U_{t+1}] \tag{14}$$

$$\zeta: \qquad \frac{D_L U_t}{D_C U_t} = -D_L f_t (1 - \psi_t) \,. \tag{15}$$

7. Key FOC's

The after-time-0 FOC's of the government with respect to K, ψ and τ are

$$\partial K: \quad \lambda_t = \beta E_t[\lambda_{t+1}(D_K f_{t+1} + \delta)]$$

$$+ \beta E_t \left[\mu_{t+1} \big(\tau_{t+1} (K_t D_{KK} f_{t+1} + D_K f_{t+1}) + \psi_{t+1} D_{KL} f_{t+1} L_{t+1} \big) \right]$$

$$+ \nu_t \beta E_t [D_C U_{t+1} (1 - \tau_{t+1}) D_{KK} f_{t+1}] + \beta E_t [\zeta_{t+1} (1 - \psi_{t+1}) D_{KL} f_{t+1}] \tag{16}$$

$$\partial \tau: \quad \nu_{t-1} D_C U_t D_K f_t = \mu_t D_K f_t K_{t-1} \tag{17}$$

$$\partial \psi: \quad \zeta_t D_L f_t = \mu_t L_t D_L f_t \tag{18}$$

8. The argument

Using the ψ and τ FOC's in the K FOC, we get

$$\lambda_t = \beta E_t[\lambda_{t+1}(D_K f_{t+1} + \delta)]$$

$$+ \beta E_t [\mu_{t+1} (K_t D_{KK} f_{t+1} + \tau_{t+1} D_K f_{t+1}) + D_{KL} f_{t+1} L_{t+1}] \,. \tag{19}$$

Note that under our assumption that f is homogeneous of degree 1, $D_K f$ is homogeneous of degree zero, which implies in turn that

$$D_{KK} f_t K_t + D_{KL} f_t L_t = 0\,. \tag{20}$$

Applying this result to (19) gives us

$$\lambda_t = \beta E_t[\lambda_{t+1}(D_K f_{t+1} + \delta)] + \beta E_t[\mu_{t+1}\tau_{t+1} D_K f_{t+1}]\,. \tag{21}$$

Because (10) and (12) are inequalities and the objective function is being maximized, not minimized, we know that the Lagrange multipliers μ and λ on these constraints are non-negative.[1] The Lagrange multipliers can be interpreted as the marginal effect on the objective function of slightly relaxing the constraint by adding a constant to the right-hand side.

In any deterministic steady state λ_t will be a constant.[2] But then comparing (13) with (21), it is easy to see that they cannot both hold in deterministic steady state if $\tau > 0$.

9. Discussion

This argument would also have worked if we had complicated the model by making f homogeneous of degree less than one. The argument is at least much harder if f is allowed to be a general concave function, and may even be impossible.

The argument made no use of the existence of government debt. If we had omitted ψ from the model, we could have derived the same conclusion, but would have had to impose a different regularity condition on the shape of the production function.[3] The role of debt would be simply to make it feasible not to match tax revenue to expenditures period by period.

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[1] Of course it is important that the constraints be set up to have the inequality read as $\leq$ before we insert the constraint into the Lagrangian. We have implicitly done so in setting up these FOC's.

[2] If we define a steady state as constant values of C, K, τ and ψ, then this in principle requires an argument, but the argument is straightforward here. Note that in deterministic steady state g_t is constant at its mean value.

[3] For example convexity of $D_K f$ as a function of K for fixed L. This condition is not a necessary consequence of concavity of f or homotheticity. It does hold for all CES production functions, for example.