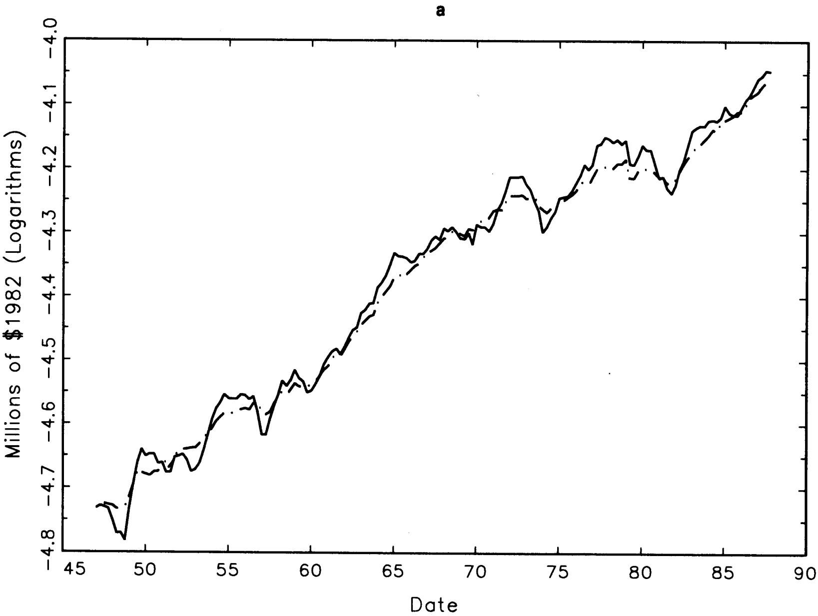
LECTURE 3: EVIDENCE ON RBC MODELS; THE GENERAL LINEAR RE MODEL

1. HOW MUCH OF THE BUSINESS CYCLE SEEMS TO BE “REAL"?

- Calibration claims
  - Historically, the Kydland-Prescott claim that the order of magnitude of observed macroeconomic fluctuations and the relative variances of consumption and investment could be reproduced in an RBC model were impressive. The result fit the assumptions neither of the Keynesians nor of the monetarists, which at the time was seen as an partition of the space macroeconomists.
  - There was no attempt to match detailed serial correlation properties of the data.
  - Certain aspects of the data were clearly not matched well:
    * the behavior of labor hours and productivity
    * the relationship between lagged interest rates and current output
  - There have been many efforts since to patch the model up. Gary Hansen’s lumpy labor supply; Eichenbaum and Christiano’s government expenditure shocks; home production models; etc.
  - Contrary to King-Rebelo, there have been quite a few previous efforts to correct the interest rate implications, and with adjustment costs on K, plus stickiness, these can be matched. The claim that no efforts of this kind have produced as “reliable a model as the RBC workhorse” is at least exaggerated. The “workhorse” is well documented to be *un*reliable, and there are examples of DSGE (dynamic, stochastic general equilibrium) models with stickiness that match sample moments as well as the “workhorse” model. (Jinill Kim, “Constructing and Estimating a Realistic Optimizing Model of Monetary Policy”, *JME*, April 2000, for example.)

2. WATSON *JPE*

Represent the data y as $y = x+u$, where x is model simulation output and u is “error". We can observe the autocovariance properties of y from the data. We can calculate the autocovariance properties of x by model simulation. “Estimate" u by minimizing $\mathrm{Var}(u)$ subject to $y = x+u$ and the known autocovariance properties of x and y. The result implies u and hence also x are exact linear functions of y. See the plots in the paper. The fit is quite bad. Most of the variation of output in US postwar recessions is attributed to the error term. Many (though not all) of the recessions are not accompanied by any negative growth in the x component.



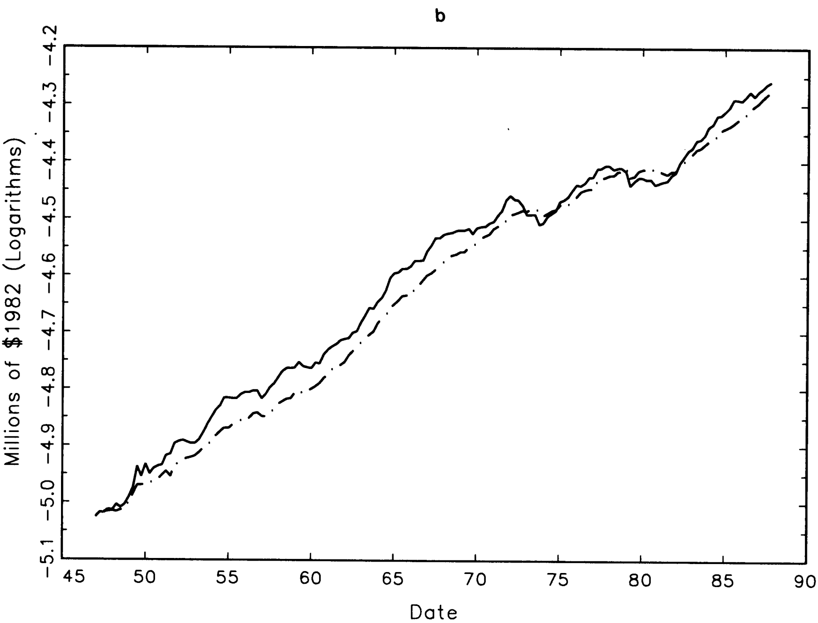
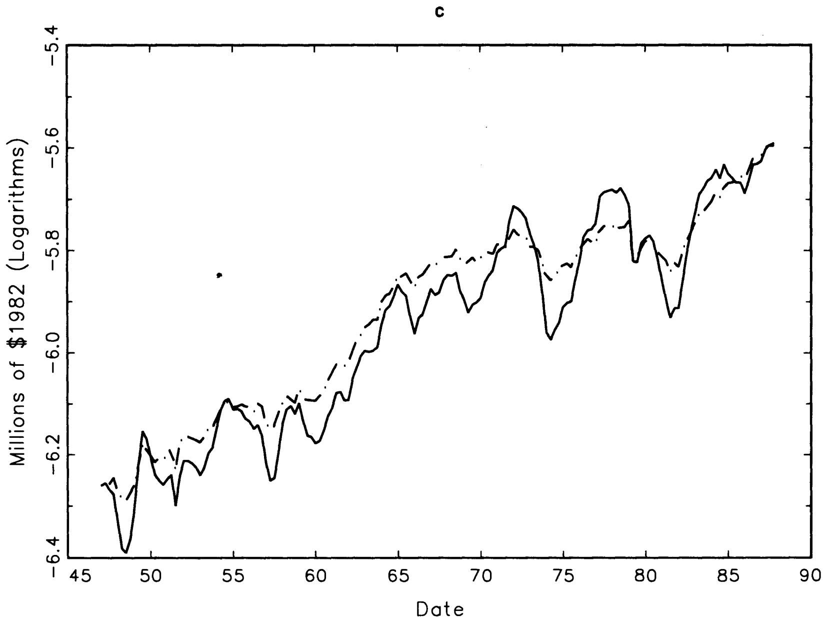


FIG. 2.—Data: *a*, output; *b*, consumption; *c*, investment; *d*, employment. Solid lines refer to U.S. data and dashed lines to realizations from the model.



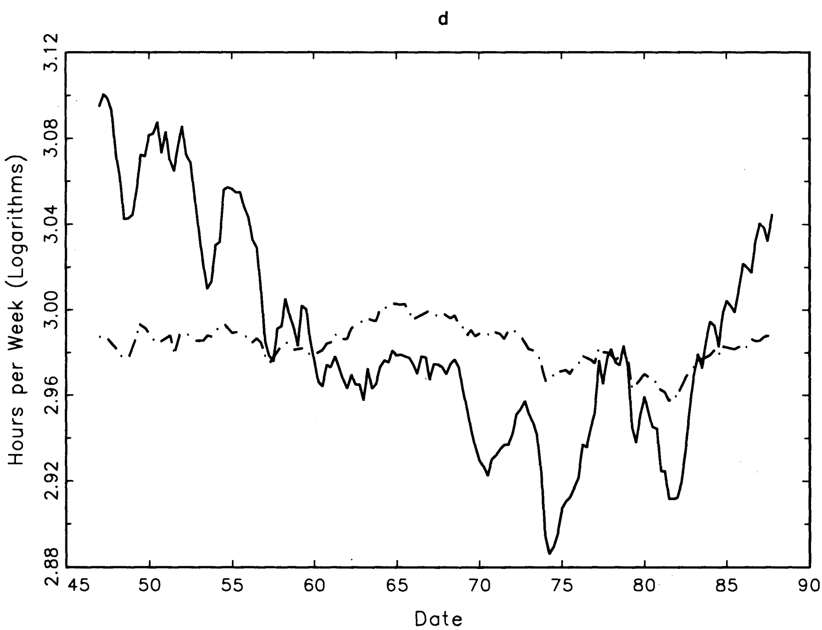


FIG. 2.—*Continued*

3. Structural VAR monetary policy literature: As an implicit critique/support of RBC models.

- These models all work off the observation (which does not depend on identifying assumptions) that surprise increases in the nominal interest rate are followed, with a delay of 6 to 24 months, by a decline in output. This, as King and Rebelo point out, has not been reproduced by standard RBC models, even with money added. (It is reproduced, though, by the Kim sticky price paper cited above.)
- These models almost all agree that the proportion of business cycle variation accounted for by monetary policy disturbances is modest, with most estimates around 10-30%. This leaves plenty of room for real business cycle mechanisms to operate.
- These models also all agree that most variation in monetary policy instruments, like the Federal Funds Rate, are accounted for by systematic reactions of monetary policy to disturbances that originate elsewhere in the economy, and that these same non-policy disturbances account for much of the business cycle. It is an open question whether, as RBC theory would suggest, the reactions of monetary policy to real disturbances play no important role in propagating them.

4. Blanchard-Quah.

- Rests on treating Aggregate Supply and Aggregate Demand as distinct and statistically independent sources of stochastic variation.
- Microeconomic supply and demand, where supply is affected by weather, behavior of a distinct class of producing agents, demand is affected by shifts in consumer tastes, are plausibly treated as statistically independent, at least as a working hypothesis.
- "Aggregate demand" depends on investment behavior, which surely is influenced by the same technology shocks that enter the marginal product of labor curve, and thus into aggregate supply. In this sense aggregate demand and supply are not as clearly distinct, as sources of variation, as are microeconomic demand and supply.
- So the B-Q finding that aggregate demand plays a very substantial role in generating fluctuations has not been treated by RBC modelers as a problem for their approach.

5. Outline

- The basic idea behind eigenvector decomposition approaches to solving linear RE models
- Canonical forms, continuous and discrete time
- What determines existence and uniqueness
- Allocating effort between yourself and the computer

6. Our most general canonical form

$$\Gamma_0 y(t) = \Gamma_1 y(t-1) + C + \Psi z(t) + \Pi\eta(t), \qquad t = 1,\ldots,T\,. \quad (1)$$

C: a vector of constants
$z(t)$: an exogenous random disturbance
$\eta(t)$: an expectational error

All we know about $\eta(t)$ is that $E_t\eta(t+1) = 0$, all t. The actual values of $\eta(t)$ have to be determined in solving the model.

Note: No $E_t x(t+1)$ terms in the system. We've replaced any such term by

$$x(t+1) - \big(x(t+1) - E_t x(t+1)\big) = x(t+1) - \eta(t+1)\,.$$

Convention: Anything dated t is known at t, i.e. $E_t x(t) \equiv x(t)$ for any x.

7. Why a Canonical Form?

- It is some work to get a model into this form. Models often have more than one lag. They often have $z(t)$ and $\eta(t+1)$ in the same equation. They often have $E_t x(t+s)$ terms with $s > 1$. But for this form, the work is modest.
- Once the model is in a canonical form, the solution set can be described automatically, by the computer.

8. Example

$$y_t = -\theta(r_t - E_t\pi_{t+1}) + E_t y_{t+1} + \varepsilon_t \quad (2)$$

$$\pi_t = \gamma y_t + \beta E_t\pi_{t+1} + \nu_t \quad (3)$$

$$\Gamma_0 = \begin{bmatrix} 1 & \theta \\ 0 & \beta \end{bmatrix}, \quad \Gamma_1 = \begin{bmatrix} 1 & 0 \\ -\gamma & 1 \end{bmatrix}, \quad \Pi = \underset{2\times 2}{I},$$

$$\Phi = \begin{bmatrix} -1 & 0 & \theta \\ 0 & -1 & 0 \end{bmatrix}.$$

9. What `gensys.m` Produces

- existence: is there any solution?
- uniqueness: is there at most one solution? (Non-existence and non-uniqueness can coexist.)
- completeness: are there as many equations as variables?

$$y(t) = \Theta_1 y(t-1) + \Theta_c + \Theta_0 z(t) + \Theta_y \sum_{s=1}^{\infty} \Theta_f^{s-1}\Theta_z E_t z(t+s) \quad (4)$$

Θ_1: `G1`
Θ_c: `C`
Θ_0: `impact`
Θ_y: `ywt`
Θ_f: `fmat`
Θ_z: `fwt`

10. Impulse responses

Impulse responses trace out the effect on the system of unit increases, lasting only one period, in elements of the z vector. If z is i.i.d., and y is stationary, the impulse responses are also the coefficients of the moving average representation for y. If z is i.i.d., the matrix of effects s periods from now on y emerging from unit increases now in z is given by the matrix $\Theta_1^s\Theta_0$, where the rows of the matrix correspond to the elements of y and the columns correspond to the elements of z that are being perturbed. When z is not i.i.d, the impulse responses depend on how expected future z's react to a change in current z, and thus can't be determined without expanding the model to describe explicitly z's serial dependence properties.

Impulse responses are often displayed by plotting the i, j'th element of this impulse response matrix as a function of s. This is the time path of the response of variable i to a unit disturbance in z. Though impulse responses contain no information not available in principle in Θ_0 and Θ_1, they are usually easier to interpret. They display “typical modes of behavior" for variables in the system and fit an “if this happens, then that happens" interpretation.

11. The Details, for a Simplified Canonical Form

- $\Gamma_0 = I$
- Stability conditions:

$$E_s\left[\phi_i y(t)\xi_i^{-t}\right] \underset{t\to\infty}{\longrightarrow} 0, \quad i = 1,\dots\infty \tag{5}$$

- Jordan decomposition

$$\Gamma_1 = P\Lambda P^{-1}$$

Λ is “almost diagonal", with “Jordan blocks" down the diagonal.

$$\Lambda = \begin{bmatrix} \Lambda_1 & 0 & \cdots & 0 \\ 0 & \Lambda_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \Lambda_m \end{bmatrix} \tag{6}$$

$$\Lambda_j = \begin{bmatrix} \lambda_j & 1 & 0 & \cdots & 0 \\ 0 & \lambda_j & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & \cdots & \lambda_j & 1 \\ 0 & 0 & \cdots & 0 & \lambda_j \end{bmatrix} \tag{7}$$

- $w(t) = P^{-1}y(t)$, so
$$w(t) = \Lambda w(t-1) + P^{-1}C + P^{-1}\big(\Psi z(t) + \Pi\eta(t)\big)$$
- Consider block j:

$$w_j(t) = \Lambda_j w_j(t-1) + P^{j\cdot}C + P^{j\cdot}\big(\Psi z(t) + \Pi\eta(t)\big)$$

- Solving backward yields

$$w_j(t) = \Lambda_j^t w_j(0) + (I-\Lambda_j)^{-1}(I-\Lambda_j^t)P^{j\cdot}C + \sum_{s=0}^{t-1}\Lambda_j^s P^{j\cdot}\left(\Psi z(t-s) + \Pi\eta(t-s)\right) \quad (8)$$

- If w_j is of length m_j, then the elements of Λ_j^t are products of polynomials in t of order at most m_j with λ_j^t, where λ_j is the diagonal element of Λ_j.
- Therefore if there is any i such that $\phi_i P^{j\cdot} \neq 0$ and $\lambda_j \geq \xi_i$, the only solution for w_j that satisfies the stability conditions is the forward solution
$$w_j(t) = (I-\Lambda_j)^{-1}P^{j\cdot}C - \sum_{s=1}^{\infty}\Lambda_j^{-s}P^{j\cdot}E_t[\Psi z(t+s)]$$
- In the special case where $E_t z(t+1) \equiv 0$, the last term drops and w_j must be a constant. But from (8), $w_j(t)$ has in this case one-step-ahead prediction error (innovation)
$$P^{j\cdot}\left(\Psi z(t) + \Pi\eta(t)\right) = 0\,. \quad (9)$$
- For every j whose root needs to be “suppressed", we get such an equation. Stacking up the corresponding $P^{j\cdot}$'s into a matrix P^u (u for “unstable"), we get
$$P^u\Psi z(t) = -P^u\Pi\eta(t)\,. \quad (10)$$
- If the space spanned by the columns of $P^u\Pi$ includes all the columns of $P^u\Psi$, then for every possible $z(t)$ we can solve for $\eta(t)$ from (10). This is the condition for existence of a solution. Notice that it depends on the idea that the $z(t)$ vectors are unrestricted.
- If the space spanned by the *rows* of $P^u\Pi$ contains all the rows of $P^s\Pi$, where P^s is the matrix formed from all the rows of P^{-1} not contained in P^u, then the value of $P^u\Pi\eta(t)$ determined by (10) also determines the value of $P^s\Pi\eta(t)$, and we have uniqueness.