

Lecture 8-9: Solution methods; Attack models*

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May 15, 2001

Solution methods: Local Expansions

- First order expansions are now common. As we have discussed, they are inadequate for some purposes, particularly for models where welfare evaluations or risk premia are of central interest.
- Second order expansions are about to become implementable with standard software packages. These too have limitations. They can generate risk premia, but not risk premia that vary with the state.
- All the local methods available now prepackaged assume that the model has a unique deterministic steady state which matches that of the first-order expansion. This condition can easily fail in a model with multiple assets.
- Higher order local expansions will, under fairly stringent regularity conditions, get more accurate as the order increases. But the accuracy will always be greatest near the point around which the expansion takes place.

Marcet-Marshall-Den Haan Parameterized Expectations

- *Two* central ideas here, that need not be tied together: Parameterize expectations functions, and evaluate accuracy based on model simulations. Originally also included the idea of minimizing a sum of squared errors criterion at each iteration.

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- Univariate growth model with CRRA utility, e.g. Equations defining equilibrium are

$$C_t + K_t = A_t K_{t-1}^\alpha + (1 - \delta) K_{t-1} \quad (1)$$

$$C_t^{-\gamma} = \beta E_t \left[C_{t+1}^{-\gamma} (\alpha A_{t+1} K_t^{\alpha-1} + 1 - \delta) \right]. \quad (2)$$

With A_t i.i.d., we can take the state to be $W_t = A_t K_{t-1}^\alpha + (1 - \delta) K_{t-1}$, and write (1) as

$$W_t = A_t (W_{t-1} - C_{t-1})^\alpha + (1 - \delta)(W_{t-1} - C_{t-1}). \quad (3)$$

Postulate

$$E_t \left[C_{t+1}^{-\gamma} (\alpha A_{t+1} K_t^{\alpha-1} + 1 - \delta) \right] = h(W_t; \theta). \quad (4)$$

For example, h might be a polynomial, with θ its vector of coefficients. Then for any given value of the parameter vector θ , we can substitute h into the right-hand side of (2), so that when combined with (3) we have a two-equation system in the two unknowns C_t and W_t . Starting from any initial value for W_0 , and given a sequence of exogenous shocks A_t , we can solve for the time paths of W and C .

- Improving the solution: Marcet/Marshall suggested estimating θ from the nonlinear regression equation (4), applying nonlinear least squares to the simulated data. Since the new θ thus estimated will imply different simulated data (even with the same A_t series), the procedure has to be iterated until it converges.

Projection Methods

- Usually, it works better to use equation-solving, rather than error-minimizing, solution algorithms. For example, one can, in the Marcet/Marshall method, replace the NLLS

adjustment of θ with solution of the equations

$$\begin{aligned}\sum_t \eta_{t+1} &= 0 \\ \sum_t \eta_{t+1}(W_t - \bar{W}) &= 0 \\ \sum_t \eta_{t+1}(W_t - \bar{W})^2 &= 0 \\ &\vdots\end{aligned}$$

where

$$\eta_t = \beta(C_{t+1}^{-\gamma}(\alpha A_{t+1} K_t^{\alpha-1} + 1 - \delta)) - C_t^{-\gamma}.$$

If we are to solve the equations, of course, there must be just as many such cross product conditions as there are free parameters in θ .

- Rather than simulating for a given θ then using iterative methods to solve the equations above, one can “open the loop”. That is, one can generate new η 's at every trial value of θ in the equation-solving process. In this way, when the equation solving has converged, the problem is solved.

Deterministic, rather than Monte Carlo, error functions

Judd, the strongest advocate of projection methods, generally suggests evaluating error according to preselected weighting functions over the state space. This idea can be applied even when we are parameterizing expectation functions. From (4) and (2) we can derive

$$C_t^{-\gamma} = h(W_t; \theta). \quad (5)$$

Using this and (3) lets us write the left-hand side of (4) as a function of W_t and exogenous disturbances alone. It is therefore possible to evaluate this left-hand side by numerical integration and compare it to $h(W_t; \theta)$. We could then solve for a θ vector that made the two sides of (4) equal at some set of values of W of the same length k as the θ vector. We could also evaluate the two sides of the equation at many values of θ (possibly an equally spaced grid) and then solve to set to zero k sensibly chosen linear combinations of the discrepancies. The advantage of such an approach is that it allows us to insure that the

approximation will be accurate in some range of W values that may be particularly interesting to us. The disadvantage is that it may be difficult to know, before solving the model, what range of W values is likely to occur frequently on a model solution path.

Speculative Attacks: The facts

- Back to the times of the gold standard, it had been observed that there were occasional “speculative attacks”, in which a country would suddenly lose a large fraction of its gold reserves.
- Similar speculative attacks occurred in fixed exchange rate regimes.
- The recent Asian Crisis wave of speculative attacks seems similar, yet the behavior of the economies and economic policy prior to the attacks and after them was different.

Theories

- The seminal idea was the work of Salant and Henderson on attacks on gold pegs. Krugman adapted the idea to fixed exchange rate systems.
- These were defined what is now known as the “first generation” class of attack models.
 - the attacks arise from an attempt to implement unsustainable policies.
 - attacks are sudden, but predictable.
- Second generation models, of which there are many
 - Governments have loss functions, instead of arbitrary policy rules.
 - The possibility of multiple equilibria, sunspots, and unpredictable attacks arises.

Our model

Why our own model?

- The principles underlying these models do not apply only to exchange rates. Any kind of potentially reversible policy commitment can generate similar phenomena.
- Most existing models focus on monetary policy, with fiscal policy in the background, while the crises always have a fiscal element, most dramatically in the recent Asian crises.

- A model without money lets us display the mechanisms of crisis with minimum technical apparatus.

Government Budget Constraint:

$$\frac{B_t}{P_t} + \frac{x_t F_t}{P_t} = R_{t-1} \frac{B_{t-1}}{P_t} + x_t R_{t-1}^* \frac{F_{t-1}}{P_t} - \tau_t .$$

Private sector:

$$1 = \beta R_t E_t \left[\frac{P_t}{P_{t+1}} \right]$$

$$1 = \beta R_t^*$$

$$1 = \beta R_t^* E_t \left[\frac{x_{t+1} P_t}{x_t P_{t+1}} \right]$$

$$x_t = P_t .$$

Solve forward using private sector conditions on returns:

$$\frac{B_t}{P_t} + \frac{x_t F_t}{P_t} = E_t \sum_{s=1}^{\infty} \beta^s \tau_{t+s}$$

Holding P fixed with reserves

Policy:

$$P_t = \bar{P} \tag{6}$$

$$F_t = -\bar{F} \tag{7}$$

$$\tau_t = \bar{\tau} < (R^* - 1) \left(\frac{B_0}{P_0} - \frac{\bar{F} x_0}{P_0} \right) \tag{8}$$

Before the crash

This policy is not sustainable, because τ is not high enough to back the amount of outstanding debt. Nonetheless, it can persist for a while. If it prevails at t and it is known that it will continue to prevail at $t + 1$, then domestic debt and foreign debt are equivalent in the eyes of investors, so $R = R^*$. Then the government budget constraint becomes

$$\frac{B_t}{\bar{P}} + F_t = R^* \left(\frac{B_{t-1}}{\bar{P}} + F_{t-1} \right) - \bar{\tau}.$$

This is an unstable linear difference equation in $B/\bar{P} + F$, the total real outstanding debt of the government. Because $\bar{\tau} < (R^* - 1)B/\bar{P}$ at the initial date, the real debt grows, approximately at the rate R^* .

Assumptions about post-crash policy

- We need to specify how the government is induced to abandon the peg and what it does after abandoning it.
- The government thinks of its reserves as supporting the price-pegging policy, so it will abandon the policy when the reserves reach zero.
- After the end of the peg, $R_t \equiv R^*$, $\tau_t \equiv \tau^*$.
- If s is the date of abandoning the peg,

$$\tau^* = \max\{\alpha_0 + \alpha_1 R_{s-1}, \bar{\tau}\} \quad (9)$$

- The logic of this setup is that when the peg is about to be abandoned, anticipations of inflation will make reserves flow out and will push up interest rates on domestic debt. For reasons we don't model explicitly, these developments trigger fiscal reform.

Determining the crash date

Assume that we know with certainty that the peg will be abandoned at s . We now calculate R_{s-1} , P_s , and τ^* as functions of B_{s-1} and R^* .

- By the usual forward-solved government budget constraint, we must have

$$\frac{B_{s-1}}{\bar{P}} = \frac{\tau^*}{R^* - 1} = \frac{\alpha_0 + \alpha_1 R_{s-1}}{R^* - 1}, \quad (10)$$

where we are assuming for now that τ^* emerges as larger than $\bar{\tau}$. This equation can obviously be solved for R_{s-1} .

- $P_s/\bar{P} = R_{s-1}/R^*$.
- from (10) our calculated τ^* exceeds $\bar{\tau}$ just when $(R^* - 1)B_{s-1}/\bar{P}$ exceeds $\bar{\tau}$. If the switch occurred before real debt reached this critical level, the induced level of primary surplus after the switch would be too high to be consistent with equilibrium.

Indeterminacy

- After debt has reached the critical level any date is a candidate for the abandonment of the peg.
- The later the date, the higher will be B_{s-1}/\bar{P} , and thus the greater the rise in R and the greater the inflation between $s - 1$ and s .
- If there is in fact an upper bound on τ^* , so that the τ^* equation only applies when it implies a τ less than the upper bound, then the peg must be abandoned before B/\bar{P} reaches a level such that the calculated τ^* exceeds the upper bound.
- Whichever date in the range between the upper and lower bound is selected by the beliefs of market participants as the “attack” date, will in fact be the date of the abandonment of the peg. Our calculations assume that the date is known with certainty, but it would be a simple extension to work out what happens if there is a known probability distribution over the attack dates.
- Sunspots. Agents in the economy could believe that the last two digits in the sunspot count on January 1st, 2000 determined the date of the crash. If this was one of the feasible dates, their belief would be confirmed.

A second-generation model

The previous model is like first generation models in the literature in that the attack arises entirely from the attempt to implement an unsustainable policy. It is more like a second-generation model in that, because it has government behavior that reacts to market variables, it has multiple equilibria.

Many second-generation models can produce a situation where a pegged exchange rate can persist, but if market participants believe it will collapse, it will collapse. In our version of such a model, there is a discrete fiscal burden, consisting of an addition b^* to the level of government debt, that arises if the price level jumps. This is meant to correspond to the situation, seen in many Asian crisis economies, where large banks or corporations were placed in financial difficulty by a devaluation, and then required fiscal bailouts. The model does not explain how the economy developed such an implicit bailout commitment; the paper by Burnside, Eichenbaum and Rebelo (and the talk by Aghion) suggest how it can happen.

- Policy**
- $\tau_t \equiv \bar{\tau}$
 - $R_t \equiv \bar{R}$
 - \bar{R} chosen consistent with constant P_t before the crisis
 - \bar{P} is the constant pre-attack P_t

Beliefs Each period there is a probability π that the attack will occur.

- What happens at s**
- $P_t = P^*$, all $t \geq s$.
 - To maintain expected real returns constant at R^* ,

$$(1 - \pi)\bar{R} + \pi\bar{R}\frac{\bar{P}}{P^*} = R^* .$$

- for all $t < s$,

$$\begin{aligned} \frac{B_t}{P_t} &= \frac{\bar{\tau}}{R^* - 1} - \sum_{s=1}^{\infty} \beta^s ((1 - \pi)^{s-1} \pi b^*) \\ &= \frac{\bar{\tau}}{R^* - 1} - \frac{\pi b^*}{R^* - 1 + \pi} . \end{aligned}$$

Determining \bar{R} • If there is no crash, price remains constant and debt grows according to

$$\frac{B_t}{\bar{P}} = \bar{R} \left(\frac{\bar{\tau}}{R^* - 1} - \frac{\pi b^*}{R^* - 1 + \pi} \right) - \bar{\tau}$$

$$= \frac{\bar{\tau}}{R^* - 1} - \frac{\pi b^*}{R^* - 1 + \pi}$$

- It is clear that we can solve this equation for a positive \bar{R} , so long as $b^* < \bar{\tau}/(R^* - 1)$, that is so long as $\bar{\tau}$ is more than large enough to back up debt in the amount b^* .

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$$\pi \frac{\bar{P}}{P^*} + 1 - \pi = \frac{R^*}{\bar{R}}.$$

From this equation we can solve for P^* .

Nature of the equilibrium Before the collapse, prices remain constant at \bar{P} , the interest rate is above R^* , and the real debt outstanding remains constant at a level lower than is consistent with $\tau = \bar{\tau}$ permanently. At the collapse, the price level jumps to a new level, creating a loss for debt holders somewhat smaller than the increase in government debt arising from b^* . But thereafter the debt remains constant. Because R_t also remains constant at $\bar{R} > R^*$, there is anticipated inflation after the collapse. This aspect of the result could be changed by supposing the government lowers R back to R^* after the collapse of the price peg.

Sunspots π can be anything the public believes it to be. If the probability of a sunspot count exceeding, say, Γ is π , and sunspot counts are i.i.d, then a public that believes that when the sunspot count exceeds Γ there will be a collapse will find its beliefs fully justified. The equilibrium we have just computed will apply to this situation.

Who is to blame

- If the public believes π to be zero, the speculative attack never occurs and $R_t \equiv R^*$. That is, the government's policy is consistent with there never being a speculative attack, so long as the public believes an attack to be impossible. In a sense, therefore, the government is not to blame for the occurrence of the attack.
- If the government could credibly commit to a different post-attack policy, it could preclude any attack and deliver a unique equilibrium. What would be required is that

the government commit to raise taxes by enough to back the increase in debt by b^* in the event of a collapse. If it were believed that this would be done, then there would be no price rise even if somehow b^* did get added to the debt burden. Since there would be no need for a price increase in order to absorb the additional debt, the price increase would never occur, and thus would never trigger the additional real debt.