Policy Game Exercise, due Friday, May 11*

Suppose the true natural rate Phillips Curve is given by

$$u_t = \bar{u} - \alpha \cdot (\pi_t - \hat{E}_{t-1}\pi_t) + \varepsilon_t \; , \tag{1}$$

where the "$\hat{E}_t$" is the public's expectation, not necessarily mathematical expectation. While some of our classroom discussion assumed only indirect control of π by the policy authority, in this exercise we assume that π_t is set directly by the policy authorities at t.

The objective of the policy authority is

$$\min_{\{u_s, \pi_s, s=0,\ldots,\infty\}} E\left[\sum_{t=0}^{\infty} \beta^t (u_t^2 + \omega\pi_t^2)\right] \; . \tag{2}$$

- Assume that $\varepsilon_t = \rho\varepsilon_{t-1} + \xi_t$, with $E_t\xi_{t+1} = 0$, all t. Also assume fully informed, rational private agents, so that $\hat{E}_t = E_t$ is in fact the mathematical expectation operator conditioned on everything known at t. Find the optimal policy under full commitment, including any special characteristics of policy at $t = 0$. How does this compare to the $\rho = 0$ case discussed in class?
- Under the same assumption on ε as in the previous part, find the no-commitment solution.
- Now assume ε_t is i.i.d. with $E\varepsilon_t \equiv 0$ and that the public has autoregressive expectations: $\hat{E}_t\pi_{t+1} = \gamma_0 + \gamma_1\pi_t$. Find the optimal government policy in this case. Discuss whether or not it makes any difference here whether the government can make believable commitments.
- Determine whether there is a self-confirming equilibrium with autoregressive expectations; that is, determine whether there are values of γ_0 and γ_1 that are consistent with an equilibrium in which the government optimizes and $\hat{E}_t\pi_{t+1} = E_t\pi_{t+1}$, all t.