Risk Sharing with Incomplete Asset Markets[*]

(1) Linearize a variant on the model from the lecture, in which there is a single asset whose dividend yield is $Y_1 + Y_2$. The budget constraint for agent 1, for example, is

$$C_1(t) + Q_t S_t = (Q_t + Y_1(t) + Y_2(t))S_{t-1} + Y_1(t) \, .$$

Expand about the $S = 0$, $C_1 \equiv C_2 \equiv \bar{Y}$ steady state, assuming $U(C_i(t))$ is of the CRR form $C^{1-\gamma}/(1 - \gamma)$. Solve the resulting linear model, verifying existence and uniqueness, either by hand or using the computer with $\beta = .95$, $\bar{Y} = 1$, and $Y_i(t)$ i.i.d. across both i and t. Solve also for the case where instead $\Delta Y_i(t)$ is i.i.d. across i and t. Discuss the difference.

(2) Show that the planner's equilibrium with equal weights, with $C_1(t) = C_2(t) \equiv (Y_1(t) + Y_2(t))/2$, cannot be supported as a competitive equilibrium in which this single risky asset is traded.

(3) Show that the equilibrium with this single asset is not the same as equilibrium with a single risk-free bond. The equilibrium for the single risk free bond is displayed in the answer to last year's "Current Account" exercise; you do not need to derive it for this exercise.