Capital Taxation Exercise*

We examine the effects of capital and labor taxation on welfare in a simple growth model that has an analytic solution.

Households solve:

$$\max C, K, L \sum_{t=0}^{\infty} \beta^t \log\big(C_t^{\theta}(1-L_t)^{(1-\theta)}\big) \tag{1}$$

subject to

$$C_t + K_t + B_t = \rho_{t-1}B_{t-1} + (1-\tau)R_tK_{t-1} + (1-\psi)w_tL_t + g \tag{2}$$

$$B_t \geq 0\,, \quad K_t \geq 0\,. \tag{3}$$

Firms solve:

$$\max_{L_t, M_t}\{M_t^{\alpha}L_t^{1-\alpha} - R_tM_t - w_tL_t\}\,. \tag{4}$$

The government budget constraint is

$$B_t = \rho_{t-1}B_{t-1} - \tau R_tK_{t-1} - \psi w_tL_t + g\,. \tag{5}$$

The market clearing conditions for labor and bonds are imposed implicitly by the use of the same "L" in the household and firm problems and the same B in the household problem and the government budget constraint. The market clearing condition for capital services is $M_t = K_{t-1}$. That is, households make the decision about how much investment to make, and they rent out "machines" to firms at the rate R_t. The goods market clearing condition is $C_t + K_t = K_{t-1}^{\alpha}L_t^{1-\alpha}$. (Because of the constant returns technology, this condition can be derived from the other equations of equilibrium.)

Note that in the problem B is real bonds, not nominal bonds, so the government does not choose the interest rate on them; once its tax and spending policy (τ and g) is fixed, it must simply offer a competitive return on the bonds it sells.

Because of the logarithmic utility function, the Cobb-Douglas production function, and the 100% depreciation rate, this model falls in the class of models for which optimal behavior implies maintaining a constant ratio $\kappa \equiv K_t/C_t$. This constancy will hold also in a model with non-zero tax rates on capital (τ) and labor (ψ). However, the optimal κ will depend on these tax policy parameters.

(a) Using transversality, show that equilibrium $\kappa = K/C$ is indeed constant in this model. Find κ as a function of the model parameters α, β, τ, and ψ.

(b) Find steady state C and L as functions of the parameters, and from that find steady state period utility $\log\bigl(C_t^{\theta}(1-L_t)^{1-\theta}\bigr)$.
(c) Find the derivatives of steady-state period utility with respect to τ and ψ at the point $\tau = \psi = 0$.
(d) Find time-0 utility (i.e. the value of the expression (1)) as a function of model parameters and initial capital stock K_{-1}.
(e) Find the derivatives of time-0 utility with respect to τ and ψ at the point $\tau = \psi = 0$. If it makes it easier, you can also assume and $K_{-1} = \bar{K}$, where $\bar{K}$ is the steady-state value of the capital stock when $\tau = \psi = 0$.
(f) Explain whatever qualitative differences you find between the response of period utility and the response of time-0 utility to tax changes.