Econ. 511b Spring 1996 C. Sims

Final Examination

Answer all three questions. They will be weighted equally in grading.

1. Suppose risk neutral agents trade a stock that pays a dividend at time t of p_t and also a one-period bond that pays out at time t the gross return r_{t-1}, which is known at t-1. Because of risk neutrality, they will insist that the expected return on the stock match that on the bond, so that if the stock price at time t is Q_t, an equilibrium condition is

$$r_t = E_t\left[\frac{Q_{t+1}+p_{t+1}}{Q_t}\right]. \tag{1}$$

Suppose further that r and p are exogenously given stochastic processes, satisfying

$$\log r_t = g \log r_{t-1} + (1-g)\log \bar{r} + e_t,\ 0<g<1 \tag{2}$$

$$\log p_t = f \log p_{t-1} + (1-f)\log \bar{p} + x_t,\ 0<f<1, \tag{3}$$

where e and x are mutually independent i.i.d. processes.

i) Display a version of this model linearized around its steady state. You may linearize in terms of logs or levels of the variables.

ii) Solve the linearized model for Q in terms of r and p, on the assumption that Q cannot grow at the rate $\bar{r}^t$ indefinitely.

iii) Are the signs of the effects of r and p on Q in this model special to the serial correlation properties of r and p that we have assumed in (2) and (3), or are they likely to hold more generally? Can you describe serial correlation properties of r and p that would imply opposite signs for their effects on Q? For this part of the question, it may be useful to consider the forward solution of equation (1)'s linearization, in which Q would depend on expected future values of r and p.

2. Consider an "AK" growth model, in which individuals maximize

$$E\left[\sum_{t=0}^{\infty} b^{t} \log C_t\right] \tag{4}$$

subject to

$$C_t + B_t + K_t = AK_{t-1}\cdot(1 - t_t) + r_{t-1}B_{t-1} \tag{5}$$

and the usual requirements that B and K remain non-negative. The tax rate t is taken as given by the agents in the economy, and it must be between zero and one. Transversality will rule out B or K going to plus infinity at a rate faster than the interest rate.

The government has an exogenously given level of required spending, G_t, which is a constant proportion g of output. It must finance this spending through taxes or debt issue. Thus its budget constraint is

$$G_t + r_{t-1}B_{t-1} = gAK_{t-1} + r_{t-1}B_{t-1} = B_t + t_t AK_{t-1}\,. \tag{6}$$

Note that in this model there is no exogenous uncertainty. We use the "E" operator in (4) only to recognize that behavior of private agents depends on their expectations about future government actions, which may in some parts of the problem below at least potentially differ from actual future government actions.

i) Derive the planner's optimum for this model. That is, find the maximum of the objective function (4) subject to the social resource constraint you can derive from (5) and (6), assuming that K can be chosen directly by the planner and taxes and debt do not exist in the economy.

ii) Suppose that at time 0 $B_{-1} = 0$ and the government must set $t_t = \bar{t}$, all $t \geq 0$. What is the optimal choice $\bar{t}^*$ of $\bar{t}$? Show that it reproduces the social optimum. We usually think of non-lump-sum taxes as "distorting". Why does a non-zero distorting tax prove to be optimal in this economy?

iii) Suppose that at time 0 $B_{-1} > 0$ and the government must set $t_t = \bar{t}$, all $t \geq 0$. Show that the government in this case cannot choose $\bar{t}$ to achieve the social optimum.

iv) Suppose that if the government truthfully announces a sequence $\{t_t\}$ of tax rates for $t = 0,\ldots\infty$, it is believed. (It does not have the option of announcing future tax rates different from what it will actually impose, but is believed when it announces actual future tax rates.) Show that in that case it can announce a $\{t_t\}$ sequence that achieves the social optimum under the assumptions of part (iii), i.e. that initial debt is positive. [This question does *not* require any complicated algebra. You should be able to see and explain the optimal policy by thinking about the structure of the problem and the nature of first-order conditions and constraints at various dates.]

Suppose the public does not believe government announcements, but instead always expects that the current tax rate will apply in all future periods. Will it then be optimal for the government to set $t_t = \bar{t}$, all $t \geq 0$, even when $B_{-1} > 0$ and thereby justify the public's beliefs?

3. Consider an economy in which there is no money, but in which government debt yields transactions services. This might actually occur with the advent of electronic cash cards, for which paying interest on "cash" might be feasible. The representative agent then faces the constraint

$$C_t(1+gv_t)+\frac{B_t - r_{t-1}B_{t-1}}{P_t}+t_t = Y_t\,, \tag{7}$$

with $v_t = P_t C_t / B_t$ and $B_t \geq 0$, and the government must satisfy the constraint

$$B_t + t_t P_t = r_{t-1}B_{t-1}\,. \tag{8}$$

The representative agent maximizes

$$E\left[\sum_{t=0}^{\infty} b^t \log C_t\right]. \tag{9}$$

The government here must be thought of as choosing t, r, and B subject to (8). Can the government determine a unique price level by setting $B_t = \overline{B}$ and $r_t = \overline{r}$? What about $r_t = \overline{r}$ and $t_t = \overline{t}$? What about $B_t = \overline{B}$ and $t_t = a + b\dfrac{B_t}{P_t}$? Explain your answers.