# Monetary \& Fiscal Policy Interactions 

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## Coverage of Lecture

1. What is price-level determination?

- introduction to the basic logic of policy interactions

2. A simple formal model

- regimes M \& F: determine $P$ \& stabilize debt
- details about nature of equilibrium \& policy interactions
- introducing maturity structure for government debt

3. A provocative proposal

- an alternative fiscal consolidation

4. Bergin's model of monetary union
5. A simple new Keynesian model

- some analytics
- including money

6. Exchange-rate \& price-level determination

- merges intertemporal approach to current account with fiscal policy


## Monetary \& Fiscal Interactions: Big Picture

- Modeling convention
- Canonical macro models assume

1. MP can and does control inflation
2. FP can and does ensure solvency
3. MP optimal or obeys Taylor-type rule

- unconstrained or "active"

2. FP takes MP \& private behavior as given and stabilizes debt

- constrained or "passive"
- This modeling convention makes sense in normal times
- embedded in textbooks (Walsh, Woodford, Galí)
- It also makes MP omnipotent \& FP trivial


## Price-Level Determination: Big Picture

- Many macroeconomists have only a narrow understanding of how the aggregate price level-and inflation-get determined
- supply \& demand in the "money market" or
- the Taylor principle for interest-rate setting
- both are equivalent to $M V=P Y$
- This narrow perspective misses important things
- role of nominal government debt
- payment of interest on reserves
- deeper understanding of "determinacy of equilibrium"
- Why focus on price-level determination?
- monetary \& fiscal policies have many other effects
- Price-level determination is first step
- study price-level determination before studying more complicated things
- We are not interested only in inflationary effects of monetary \& fiscal policies


## Monetary \& Fiscal Interactions: Big Picture

- The "money-only" modeling convention a stretch since 2008
- What have policies actually been doing?

1. MP at or near lower bound-policy rates $<0$ in

Europe \& Japan
2. FP bouncing between stimulus \& austerity

1. Central banks aggressively pursuing growth

- thrown Taylor principle out the window
- undertaking variety of large-scale asset purchases

2. Fiscal policies

- 2008-2009: significant, explicitly temporary, stimulus
- 2010-2011: declare victory \& consolidate
- since 2012: varies, but generally little to help recovery
- How can such policies anchor monetary expectations on inflation target?
- How can such policies anchor fiscal expectations on debt stabilization?


## Monetary \& Fiscal Interactions: Big Picture

- Need to understand implications of policy interactions that deviate from convention
- Short-run reasons:
- European countries \& Japan mired in "low-flation"; many advanced \& emerging economies have high government debt
- Ubiquitous tradeoff between macroeconomic stabilization \& fiscal sustainability
- What are effect of fiscal policy when MP pegs rate?
- What are the fiscal implications of exploding CB balance sheets?
- Long-run reasons:
- Aging populations \& unfunded old-age benefits
- Huge uncertainty about future fiscal policies
- Conventional modeling cannot address these issues
- assumes away the problems


## Messages

1. Effects of monetary policy-open-market operations- depend on the sense in which fiscal policy is "held constant"
2. Effects of fiscal policy-bond-financed tax cuts-depend on the sense in which monetary policy is "held constant"
3. MP cannot uniquely determine inflation; FP can
4. MP can uniquely determine bounded inflation-if FP cooperates
5. If FP does not cooperate, MP cannot affect economy in usual ways
6. Without credible, enforceable fiscal rules that anchor expectations on appropriate FP behavior, fiscal disturbances always affect economy

## Perspectives on the Price Level

An Accounting Perspective

- Nominal government bonds
- asset to private sector
- liability to government
- Real primary surpluses (think of as taxes)
- liability to private sector
- asset to government
- Real assets must equal real liabilities
- Price level must adjust to equalize
- real value of debt outstanding \&
- expected present value of surplus stream
- Price-level determination arises from the margin between nominal asset \& real backing for the asset


## Perspectives on the Price Level

An Economic Perspective

- Nominal government bond holdings a source of wealth to private sector
- Expected stream of taxes is an obligation of private sector
- If bonds rise, but PV taxes does not. . .
- households feel wealthier
- try to convert higher nominal wealth into higher consumption path
- increases demand for goods-"aggregate demand"
- raises price level
- Wealth effects lie at the heart of price-level determination


## Perspectives on the Price Level

An Asset-Pricing Perspective

- Value of any asset equals expected PV of cash flows
- Primary surpluses-exclusive of interest payments-are cash flows underlying government debt
- Higher primary surpluses raise value of debt
- induces households to substitute out of buying goods \& into holding bonds
- raises price of bonds in terms of goods
- reduces price level
- Higher real backing for bonds increases their value \& drives down price level


## Two Kinds of Government Debt

- Distinction between real \& nominal debt is critical

1. Real debt: denominated in "goods"

- arises whenever debt is in units whose quantity the government cannot control
- indexed to inflation; foreign currency; gold
- in most countries today only small fraction of debt is real
- indexed debt is like debt under the Gold Standard, where governments did not control the price level
- a claim to goods in the future
- government must acquire those goods to honor obligations
- can acquire goods through taxes or money creation (seigniorage)
- if it cannot acquire the goods, default only option


## Two Kinds of Government Debt

2. Nominal debt: denominated in home currency ("dollars")

- arises whenever debt is in units whose supply the government can control
- vast majority of government debt is of this kind
- a claim to "dollars" in the future
- government need not be able to acquire goods
- it can print new "dollars" to reduce market value of debt ("dollars" can be new debt instruments-not necessarily currency)
- default less likely
- This distinction carries important policy implications


## Two Kinds of Government Debt

- E.A. countries don't control their monetary policy
- to each country, debt in euros is real debt
- Default on real debt more likely: euro rates embed default premium

|  | Debt/GDP | 10-year yield |
| :--- | :---: | :---: |
| Real |  |  |
| Greece | 159 | 22.5 |
| Italy | 123 | 5.5 |
| Spain | 85 | 5.9 |
| Germany | 80 | 1.5 |
| Nominal |  |  |
| Japan | 237 | 0.8 |
| U.K. | 86 | 1.9 |
| U.S. | 102 | 1.8 |

General government debt as percentage of GDP \& 10-year government bond yield in 2012. Sources: ECB, Eurostat, IMF

## The Model

- Endowment economy at the cashless limit; incomplete financial markets, only one-period nominal debt
- Representative household maximizes

$$
E_{0}\left\{\sum_{t=0}^{\infty} \beta^{t} U\left(C_{t}\right)\right\}
$$

subject to sequence of flow budget constraints

$$
P_{t} C_{t}+P_{t} \tau_{t}+Q_{t, t+1} B_{t}=P_{t} Y_{t}+P_{t} z_{t}+B_{t-1}
$$

given $B_{-1}>0$

- $Q_{t, t+1}$ : nominal price at $t$ of a bond that pays $\$ 1$ at $t+1$
- $m_{t, t+1}$ : one-period stochastic discount factor
- $Q_{t, t+1}=E_{t}\left[\frac{P_{t}}{P_{t+1}} m_{t, t+1}\right]$ : no-arbitrage condition
- Nominal interest rate, $R_{t}: \frac{1}{R_{t}}=Q_{t, t+1}$


## The Model

- HH choices satisfy transversality \& no-Ponzi conditions

$$
\lim _{T \rightarrow \infty} E_{t}\left[m_{t, T} \frac{B_{T}}{P_{T}}\right]=0
$$

- it is not optimal for HH to over- or under-accumulate assets
- These imply the HH's real intertemporal b.c.

$$
\begin{aligned}
& E_{t} \sum_{j=0}^{\infty} m_{t, t+j} C_{t+j}=\frac{B_{t-1}}{P_{t}}+E_{t} \sum_{j=0}^{\infty} m_{t, t+j}\left(Y_{t+j}-s_{t+j}\right) \\
s_{t} \equiv & \tau_{t}-z_{t}
\end{aligned}
$$

- $m_{t, t+j} \equiv \prod_{k=0}^{j} m_{t, t+k}$ is real discount factor, $m_{t, t}=1$
- Constant endowment: $m_{t, t+j} \equiv \beta^{j}$


## The Model

- Impose equilibrium, $C_{t}=Y$, and TVC to get two eqm conditions

$$
\begin{aligned}
\frac{1}{R_{t}} & =\beta E_{t} \frac{P_{t}}{P_{t+1}} \equiv \beta E_{t} \frac{1}{\pi_{t+1}} \\
\frac{B_{t-1}}{P_{t}} & =\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j}
\end{aligned}
$$

(Fisher relation)
(Bond valuation)
$s_{t} \equiv \tau_{t}-z_{t}\left(\right.$ We assume $\left.0<E_{t} P V(s)<\infty\right)$

- Price sequence $\left\{P_{t}\right\}$ must satisfy these to be an eqm (markets clear \& HH's optimization problem solved)
- Without additional restrictions from policy behavior, there are many possible eqm $\left\{P_{t}\right\}$ sequences


## The Model

- Cashless economy: $1 / P_{t}$ is goods price of nominal bond; $1 / R_{t}$ is dollar price of bond
- Ad hoc policy rules

$$
\begin{aligned}
\text { MP: } & \frac{1}{R_{t}}=\frac{1}{R^{*}}+\alpha\left(\frac{1}{\pi_{t}}-\frac{1}{\pi^{*}}\right)+\varepsilon_{t}^{M} \\
\text { FP: } & s_{t}=s^{*}+\gamma\left(\frac{1}{R_{t-1}} \frac{B_{t-1}}{P_{t-1}}-\frac{b^{*}}{R^{*}}\right)+\varepsilon_{t}^{F}
\end{aligned}
$$

- Combine rules with Euler equation \& government budget constraint to yield dynamic equations in $\nu_{t} \equiv 1 / \pi_{t}$ and $b_{t} \equiv B_{t} / P_{t}$


## Separate Dynamics?

$$
\begin{aligned}
E_{t}\left(\nu_{t+1}-\nu^{*}\right) & =\frac{\alpha}{\beta}\left(\nu_{t}-\nu^{*}\right)+\frac{1}{\beta} \varepsilon_{t}^{M} \\
E_{t}\left(\frac{b_{t+1}}{R_{t+1}}-\frac{b^{*}}{R^{*}}\right) & =\left(\beta^{-1}-\gamma\right)\left(\frac{b_{t}}{R_{t}}-\frac{b^{*}}{R^{*}}\right)-E_{t} \varepsilon_{t+1}^{F}
\end{aligned}
$$

- Appears as if
- inflation dynamics driven only by MP through $\left(\alpha, \varepsilon_{t}^{M}\right)$
- debt dynamics driven only by FP through $\left(\gamma, \varepsilon_{t}^{F}\right)$
- Regime M: $|\alpha / \beta|>1 \&\left|\beta^{-1}-\gamma\right|<1$
- Regime F: $|\alpha / \beta|<1 \&\left|\beta^{-1}-\gamma\right|>1$
- In either regime, in equilibrium policies interact to determine inflation \& stabilize debt


## Two Tasks of Policy

- Monetary \& fiscal policy have two tasks: (1) control inflation; (2) stabilize debt
- Two different policy mixes that can accomplish these tasks

Regime M: conventional assignment-MP targets inflation; FP targets real debt (called active MP/passive FP)

Regime F: alternative assignment-MP maintains value of debt; FP controls inflation (called passive MP/active FP)

- Regime M: conventional "monetarist/new Keynesian"
- Regime F: alternative "fiscal theory"


## Regime M

- Bounded solution: only MP shocks cause $\pi_{t} \neq \pi^{*}$

$$
\begin{aligned}
\nu_{t} & =\nu^{*}-\frac{1}{\alpha} \sum_{j=0}^{\infty}\left(\frac{\beta}{\alpha}\right)^{j} E_{t} \varepsilon_{t+j}^{M} \\
\frac{1}{R_{t}} & =\frac{1}{R^{*}}-\sum_{j=1}^{\infty}\left(\frac{\beta}{\alpha}\right)^{j} E_{t} \varepsilon_{t+j}^{M}
\end{aligned}
$$

- Equilibrium inflation appears to depend only on monetary policy
- policy parameter: $\alpha$
- policy shock: $\varepsilon_{t}^{M}$
- Fiscal policy does not seem to matter
- Delivers Friedman: "inflation is always and everywhere a monetary phenomenon"


## Regime M

-What is the fiscal backing for monetary policy?

- passive FP: $\gamma>\beta^{-1}-1$ (net real interest rate) covers debt service \& retires debt
- assume $\varepsilon_{t}^{M} \sim$ i.i.d. and $\varepsilon_{t}^{M}>0$
- raises $P_{t}$, reduces real value of outstanding bonds, $B_{t-1} / P_{t}$, \& market value of debt, $B_{t} / R_{t} P_{t}$
- if $s_{t+j}$ unchanged, reduced real debt gets passed into lower nominal debt growth
- eventually, people will realize their wealth has declined and reduce their demand for goods
- lower demand will reduce price level, counteracting MP
- if lower real debt is backed by lower $s_{t+j}$, fiscal policy eliminates the negative wealth effect
- this fiscal backing permits monetary policy to control inflation in the usual way
- this is the definition of passive fiscal policy


## Regime M

- Friedman's adage requires an addendum: "inflation is always and everywhere a monetary phenomenon, so long as FP eliminates wealth effects of policy"
- As Tobin put it: "Ricardian equivalence is fundamental, perhaps indispensable, to monetarism"
- Consider an i.i.d. tax cut: $\varepsilon_{t}^{F}<0$
- has no effect on inflation or nominal interest rate
- financed by higher $B_{t} \Rightarrow$ higher $b_{t}$
- passive FP: higher future $\left\{s_{t+j}\right\}$
- $b_{t} \rightarrow b^{*}$
- delivers neutrality of tax-debt swaps
- Passive FP achieves two things:

1. Stabilizes real debt
2. Provides appropriate fiscal backing to MP

## Regime M Equilibrium

- Unique bounded equilibrium inflation rate
- Stable process for government debt
- But. . . also a continuum of equilibria with

$$
\lim _{T \rightarrow \infty} \pi_{T}=\infty
$$

- Neither MP nor private behavior rules out equilibria with $\pi_{t}=\infty$
- This (minor?) anomaly or embarrassment can be resolved only by fiscal policy


## Regime M's Explosive Solutions

- Examine perfect foresight; generalize policy rule

$$
\begin{aligned}
R_{t} & =\beta^{-1} \pi_{t+1} \\
R_{t} & =\tilde{\Phi}\left(\pi_{t}\right)
\end{aligned}
$$

- Solution satisfies non-linear difference equation

$$
\pi_{t+1}=\Phi\left(\pi_{t}\right)
$$

- Two steady states: $\pi^{*}$ and $\pi_{L}$
- $\pi_{L}$ are zero lower bound for nominal interest rate


## Regime M's Explosive Solutions



Indeterminacy of steady state and dynamic path

## Regime $F$

- Take case of exogenous surpluses, $\gamma=0$
- Solve for market value of debt, $b_{t} / R_{t}$, \& use GBC

$$
P_{t}=\frac{B_{t-1}}{(1-\beta)^{-1} s^{*}+\sum_{j=0}^{\infty} \beta^{j} E_{t} \varepsilon_{t+j}^{F}}
$$

- only FP—including $B_{t-1}$ —appears to matter
- Increase in current or expected transfers
- no offsetting taxes expected, household wealth rises
- lower expected path of surpluses reduces "cash flows," lowers value of debt
- individuals shed debt in favor of consumption, raising aggregate demand
- higher current \& future inflation and economic activity
- long bonds shift inflation into future
- Demand for debt $\Leftrightarrow$ aggregate demand


## Regime $F$

- How does monetary policy stabilize debt?

$$
E_{t}\left(\frac{b_{t+1}}{R_{t+1}}-\frac{b^{*}}{R^{*}}\right)=\frac{1}{\beta}\left(\frac{b_{t}}{R_{t}}-\frac{b^{*}}{R^{*}}\right)
$$

- debt dynamics: $b_{t} / R_{t}$ expected to grow at $\beta^{-1}$
- this appears to violate the transversality condition, which implies cannot be an equilibrium
- MP stabilizes $b_{t} / R_{t}$ by preventing interest payments from exploding
- appears as surprises in $P_{t}$ that revalue debt
- MP accomplishes this through its interest-rate policy
- Show this for i.i.d. $\varepsilon^{F} \Rightarrow b_{t+j} / R_{t+j}$ constant


## Regime $F$

- Use expression for $\nu_{t}$ in MP rule

$$
\frac{1}{R_{t}}-\frac{1}{R^{*}}=\frac{\alpha}{\beta}\left(\frac{\beta(1-\beta)^{-1} s^{*}+\beta \varepsilon_{t}^{F}}{b_{t-1}}-\frac{1}{R^{*}}\right)+\varepsilon_{t}^{M}
$$

- Fiscal expansion: $\varepsilon_{t}^{F}<0$
- MP reduces $1 / R_{t}$ by $\left(\alpha / b_{t-1}\right) \varepsilon_{t}^{F}$ to fight inflation
- i.i.d. shock $\Rightarrow b_{t} / R_{t}=b^{*} / R^{*}$
- at $t+1$, interest rate obeys

$$
\frac{1}{R_{t+1}}-\frac{1}{R^{*}}=\frac{\alpha}{\beta}\left(\frac{1}{R_{t}}-\frac{1}{R^{*}}\right)
$$

- if MP were active, $\alpha / \beta>1,1 / R_{t}$ diverges
- exploding paths due to wealth effects from ever-growing interest payments to bond holders
- higher wealth $\Rightarrow$ higher $\pi_{t+1} \Rightarrow$ higher $R_{t+1}$ etc.
- active MP converts stable fiscal inflation into explosive inflation


## Regime $F$

- Monetary policy rule implies

$$
\frac{1}{R_{t}}-\frac{1}{R^{*}}=\frac{\alpha}{\beta}\left(\frac{\beta(1-\beta)^{-1} s^{*}+\beta \varepsilon_{t}^{F}}{b_{t-1}}-\frac{1}{R^{*}}\right)+\varepsilon_{t}^{M}
$$

- fiscal expansion, $\varepsilon_{t}^{F}<0$, financed with higher $B_{t}$
- if MP pegs $R_{t}=R^{*}$, it fixes future inflation by fixing interest payments that fiscal expansion would raise
- MP contraction, $\varepsilon_{t}^{M}<0$, lowers $1 / R_{t}$, raises interest payments
- FP does not raise surpluses to eliminate this wealth effect
- if future inflation were not to rise, nominal debt would grow
- raises wealth still more, so eventually inflation must increase
- these different MP impacts arise from the different "fiscal backing" of MP


## An Equilibrium Condition

$$
\frac{B_{t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[s_{t+j}\right]
$$

- In Regime M (active monetary/passive fiscal). . .
- MP delivers equilibrium inflation process
- taking inflation as given, FP must choose compatible surplus policy
- "compatible" means: stabilizes debt \& passively provides appropriate fiscal backing
- imposes restrictions on $E_{t} P V(s)$


## An Equilibrium Condition

$$
\frac{B_{t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[s_{t+j}\right]
$$

- In Regime F (passive monetary/active fiscal)...
- FP delivers unique equilibrium price process
- taking inflation as given, MP must choose compatible interest rate policy
- "compatible" means: stabilizes debt
- imposes restrictions on $P_{t}$ (\& on MP, if price level to remain stable)


## More on the Equilibrium Condition

$$
\frac{B_{t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[s_{t+j}\right]
$$

- Ubiquitous: holds in any model, in any regime
- cannot be used to "test" for regime
- It is not an "intertemporal government budget constraint"
- have imposed market clearing, Euler equations, transversality (from private behavior)
- Government is not restricted to choose $\left\{s_{t}\right\}$ to satisfy it for any $\left\{P_{t}\right\}$ (but it is free to do so)
- Cochrane calls it a "debt valuation equation"
- with only one-period debt, $B_{t-1} / P_{t}$ is market value of debt


## Primer on Monetary-Fiscal Interactions

- Unpleasant monetarist arithmetic
- economy hits the fiscal limit
- surpluses unresponsive to debt
- seigniorage adjusts to stabilize debt
- produces high \& volatile inflation
- Many countries have guarded against this
- central bank independence
- clear mandate to control inflation-e.g., inflation targeting
- Designed to force FP to be passive
- Will focus on second way Regime F can arise


## Primer on Monetary-Fiscal Interactions

- Monetary \& fiscal policy have two tasks: (1) control inflation; (2) stabilize debt
- Beautiful symmetry: two different policy mixes that can accomplish these tasks

Regime M: conventional assignment-MP targets inflation; FP targets real debt (called active MP/passive FP)

Regime F: alternative assignment-MP maintains value of debt; FP controls inflation (called passive MP/active FP)

- Regime M: normal state of affairs
- Regime F: can arise in an era of fiscal stress
- Regime F arises in two ways

1. fiscal theory of the price level
2. Sargent \& Wallace's unpleasant monetarist arithmetic

## Why Fiscal Theory $\neq$ Unpleasant Arithmetic

- Equilibrium conditions for nominal and real debt

Nominal: $B_{t-1}=P_{t} \sum_{j=0}^{\infty} \beta^{j} E_{t}\left[\tau_{t+j}-z_{t+j}+\frac{M_{t+j}-M_{t+j-1}}{P_{t+j}}\right]$

$$
\text { Real: } v_{t-1}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[\tau_{t+j}-z_{t+j}+\frac{M_{t+j}-M_{t+j-1}}{P_{t+j}}\right]
$$

- Hypothetical increase in $P_{t}$, all else fixed
- raises nominal backing: support more nominal debt with no change in surpluses or seigniorage
- lowers real backing: reduces seigniorage revenues
- Fiscal Theory is not about seigniorage: if $M / P$ tiny, higher $P_{t}$ raises backing of nominal debt but not of real debt
- Unpleasant Arithmetic is about seigniorage: growing real debt requires growing seigniorage \& inflation


## Role of Debt Maturity Structure: I

- Allow one- and two-period zero-coupon nominal bonds: $B_{t}(t+1), B_{t}(t+2)$; equilibrium condition is

$$
\frac{B_{t-1}(t)}{P_{t}}+\beta B_{t-1}(t+1) E_{t} \frac{1}{P_{t+1}}=\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j}
$$

- MP determines the timing of inflation
- stabilize expected inflation: forces adjustment in $P_{t}$
- lean against current inflation: forces adjustment in $E_{t}\left(1 / P_{t+1}\right)$
- tradeoff depends on maturity structure,

$$
B_{t-1}(t+1) / B_{t-1}(t)
$$

- shorter average maturity $\Rightarrow$ need larger $\Delta E_{t}\left(1 / P_{t+1}\right)$ to compensate for given $\Delta\left(1 / P_{t}\right)$
- Message: MP not impotent, but it cannot control both actual \& expected inflation


## Role of Debt Maturity Structure: II

- Allow a consol: perpetuity that pays $\$ 1$ each period
- Government budget constraint

$$
\frac{Q_{t} B_{t}}{P_{t}}+s_{t}=\frac{\left(1+Q_{t}\right) B_{t-1}}{P_{t}}
$$

- Asset-pricing relation, in equilibrium

$$
Q_{t}=\beta E_{t} \frac{P_{t}}{P_{t+1}}\left(1+Q_{t+1}\right)=\sum_{j=1}^{\infty} \beta^{j} E_{t} \frac{P_{t}}{P_{t+j}}
$$

- Central bank controls $R_{t}: 1 / R_{t}=P_{S t}=\beta E_{t}\left(P_{t} / P_{t+1}\right)$
- Intertemporal equilibrium condition

$$
\frac{\left(1+Q_{t}\right) B_{t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j}
$$

- FP determines the present value of inflation; MP determines the timing of inflation


## Role of Debt Maturity Structure: II

$$
\begin{aligned}
Q_{t}=E_{t} \sum_{j=0}^{\infty}\left(\frac{1}{\prod_{i=0}^{j} R_{t+i}}\right) & =E_{t} \sum_{j=1}^{\infty} \beta^{j}\left(\frac{1}{\prod_{i=1}^{j} \pi_{t+i}}\right) \\
\frac{\left(1+Q_{t}\right) B_{t-1}}{P_{t}} & =\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j}
\end{aligned}
$$

- Any path of $\left\{P_{t}\right\}$ consistent with these conditions is an equilibrium
- By choosing a (constrained) path for $\left\{R_{t}\right\}$, MP determines when inflation occurs
- Consider two pegged paths for $R_{t} — \dagger \& *$ - with $R^{\dagger}>R^{*} \Rightarrow Q^{\dagger}<Q^{*}$
- $\pi_{t}^{\dagger}<\pi_{t}^{*}$ but future $\pi^{\dagger}>$ future $\pi^{*}$
- a higher nominal rate lowers current inflation, but raises future inflation


## Role of Debt Maturity Structure: III

- Zero-coupon bonds
- Write government's flow constraint as

$$
B_{t-1}(t)-\sum_{j=1}^{\infty} Q_{t}(t+j)\left[B_{t}(t+j)-B_{t-1}(t+j)\right]=P_{t} s_{t}
$$

- Impose equilibrium on asset-pricing relation

$$
Q_{t}(t+j)=\beta^{j} E_{t} \frac{P_{t}}{P_{t+j}}
$$

- Combine these

$$
\frac{B_{t-1}(t)}{P_{t}}-\sum_{j=1}^{\infty} \beta^{j} E_{t} \frac{1}{P_{t+j}}\left[B_{t}(t+j)-B_{t-1}(t+j)\right]=s_{t}
$$

## Role of Debt Maturity Structure: III

$$
\frac{B_{t-1}(t)}{P_{t}}-\sum_{j=1}^{\infty} \beta^{j} E_{t} \frac{1}{P_{t+j}}\left[B_{t}(t+j)-B_{t-1}(t+j)\right]=s_{t}
$$

- Suppose govt neither issues new debt nor repurchases outstanding debt, so

$$
\begin{array}{r}
B_{t-1}(t+j)=B_{t}(t+j)=B_{t-1}(t), j>0 \\
P_{t}=\frac{B_{t-1}(t)}{s_{t}}
\end{array}
$$

- Future deficits don't matter (constant debt $\Rightarrow$ no link between value of debt today \& future surpluses)
- Inflation occurs only when surplus realized
- Bond prices reflect $E_{t} s_{t+j}$ which changes $E_{t}\left(1 / P_{t+j}\right)$

$$
Q_{t}(t+j)=\beta^{j} E_{t} \frac{P_{t}}{P_{t+j}}
$$

## A Provocative Proposal

- Many countries face substantial fiscal consolidation
- U.K. and U.S. in 2012
- U.K. net national debt about 70\% GDP
- U.S. federal debt about 80\% of GDP
- If debt is "risk-free" then bondholders must expect primary surpluses with present value consistent with current debt-GDP ratio
- Suppose consolidation aims to reduce ratio from 80\% to 60\%
- Two steps involved

1. put current primary deficits on path to primary surpluses
2. converge to long-run primary surpluses consistent with $60 \%$ ratio

## A Provocative Proposal

- Regime M \& Regime F consolidations look very different
- Regime M Consolidation

1. raise taxes \& cut spending to convert deficit to surplus
2. continue to raise surplus to retire current debt toward 60\%
3. reduce surplus to level consistent with long-run debt target

- Regime F Consolidation

1. raise taxes \& cut spending to convert deficit to surplus
2. reduce surplus to level consistent with long-run debt target

- Regime F does not require higher surpluses to retire debt


## Hypothetical Conventional Consolidation

- To achieve the long-run reduction in debt, must
- substantially cut spending or raise taxes to overshoot surplus target
- can overshoot for decades
- then can gradually reduce primary surpluses
- These short-run adjustments will certainly slow economic growth
- slower growth will automatically reduce revenues \& increase expenditures
- these impacts are not reflected in the graph
- This is what many European countries have been doing, bringing new recessions
- What are the welfare costs of conventional consolidation?


## Hypothetical Conventional Consolidation




Paths of Primary Surplus \& Debt: Debt-GDP from 80\% to 60\% Surpluses Must Overshoot Long-Run Target

## Alternative Fiscal Consolidations

- Conventional consolidation takes inflation off table
- What can inflation do?
- government debt is nominal \& long-term
- current or future inflation devalues debt
- can avoid overshooting surplus target
- requires less fiscal adjustment
- But wait. . . there's more
- if monetary policy prevents nominal rates from rising with inflation-as it has the past 4 years
- then real interest rates fall
- stimulates consumption \& aggregate demand
- Alternative consolidation can avoid retarding growth
- What are the welfare costs of alternative consolidation?


## Hypothetical Alternative Consolidation

Primary Surplus



Paths of Primary Surplus \& Debt: Debt-GDP from 80\% to 60\%

## Illustrative Model of Inflation Determination

- Endowment economy with infinitely-lived agents, at cashless limit
- Long-term nominal bonds, $B_{M t}$, sell at price $P_{M t}$
- bond issued at $t$ pays $\rho^{j}$ dollars at $t+j+1$
- average duration of bond: $(1-\beta \rho)^{-1}$
- $\rho=0$ : all bonds 1 period
- FP: chooses primary surplus, $s_{t}$
- MP: chooses 1-period nominal interest rate, $R_{t}$
- Debt Management: chooses average maturity, $\rho$
- Equilibrium: $c_{t}=y$ for all $t$


## Government Behavior

- Government's choices of $\left\{R_{t}, s_{t}, B_{M t}\right\}$ and $\rho$ satisfy

$$
\frac{P_{M t} B_{M t}}{P_{t}}+s_{t}=\frac{\left(1+\rho P_{M t}\right) B_{M t-1}}{P_{t}}
$$

- For now, government not optimizing
- posit ad hoc—but typical—rule
- on agenda: compute welfare consequences of alternative consolidation schemes
- Government's choices constrained by conditions for equilibrium
- market clearing
- household's first-order conditions
- household's transversality condition: optimal behavior limits growth rate of government debt


## Asset-Pricing Relations

$$
\begin{aligned}
\frac{1}{R_{t}} & =\beta E_{t}\left(\frac{1}{\pi_{t+1}}\right) \\
P_{M t} & =\frac{1}{R_{t}} E_{t}\left(1+\rho P_{M t+1}\right)
\end{aligned}
$$

These imply

$$
\begin{aligned}
P_{M t} & =\beta \sum_{j=0}^{\infty}(\beta \rho)^{j} E_{t}\left(\prod_{i=0}^{j} \frac{1}{\pi_{t+i+1}}\right) \\
& =\sum_{j=0}^{\infty} \rho^{j} E_{t}\left(\prod_{i=0}^{j} \frac{1}{R_{t+i}}\right)
\end{aligned}
$$

## An Equilibrium Condition

- Imposing equilibrium, asset-pricing relations, transversality

$$
\begin{equation*}
\frac{\left(1+\rho P_{M t}\right) B_{M t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j} \tag{IEC}
\end{equation*}
$$

- In conventional consolidation...
- MP unconstrained: determines equilibrium $\left\{P_{t}\right\} \Rightarrow\left\{P_{M t}\right\}$
- FP constrained: chooses $\left\{s_{t}\right\}$ to satisfy (IEC)
- In the alternative consolidation...
- FP unconstrained: determines equilibrium $\left\{P_{t}, P_{M t}\right\}$
- MP constrained: determines timing of inflation


## Thought Experiment

- Take path of $\left\{s_{t}\right\}$ for 2012-2022 from Congressional Budget Office "Budget Projections," March 2012
- conventional consolidation: $s_{t}$ for 2023 \& 2024 increases by $1 \%$ each year
- alternative consolidation: $s_{t}$ reaches long-run target early
- Debt-output, $P_{M t} B_{M t} / P_{t}$
- initial: $80 \%$
- target: 60\%
- Model calibration

1. real interest rate $2 \%$
2. initial inflation $2 \%$
3. vary average maturity

## Conventional Consolidation

- MP obeys

$$
\frac{1}{R_{t}}=\frac{1}{R^{*}}+\alpha\left(\frac{1}{\pi_{t}}-\frac{1}{\pi^{*}}\right)
$$

- Combine with Euler equation

$$
E_{t}\left(\frac{1}{\pi_{t+1}}-\frac{1}{\pi^{*}}\right)=\frac{\alpha}{\beta}\left(\frac{1}{\pi_{t}}-\frac{1}{\pi^{*}}\right)
$$

- Unique bounded solution when $\alpha>\beta$ is

$$
\pi_{t}=\pi^{*} \text { for all } t
$$

## Conventional Consolidation

- After CBO projection period, $s_{t}$ obeys

$$
s_{t}=s^{*}+\gamma\left(P_{M t-1} b_{M t-1}-P_{M}^{*} b_{M}^{*}\right)
$$

- Impose the Euler equation

$$
E_{t-1}\left(\frac{1+\rho P_{M t}}{\pi_{t}}\right)=\frac{1}{\beta} P_{M t-1}
$$

on government's flow constraint and substitute $s$ rule

$$
E_{t}\left(\frac{P_{M t+1} b_{M t+1}-P_{M}^{*} b_{M}^{*}}{P_{t+1}}\right)=\left(\beta^{-1}-\gamma\right)\left(\frac{P_{M t} b_{M t}-P_{M}^{*} b_{M}^{*}}{P_{t}}\right)
$$

- $\gamma>\beta^{-1}-1$ stabilizes debt, ensuring (IEC) holds
- Overshooting: $P_{M t-1} b_{M t-1}>P_{M}^{*} b_{M}^{*} \Rightarrow s_{t}>s^{*}$


## Conventional Consolidation

- With MP aggressively targeting inflation. . .
- inflation cannot be used to reduce value of debt
- consolidation requires surplus to overshoot long-run target
- higher surpluses retire debt to achieve 60\% target
- In reasonable model, where taxes distort \& government spending affects demand...
- during overshooting, output will fall
- choice of $\gamma$ determines speed of adjustment
- higher $\gamma$ amplifies overshooting, exacerbating economic downturn
- lower $\gamma$ prolongs adjustment period, keeping output persistently weak
- Should we take inflation off the table?


## Alternative Consolidations

- FP sets $\left\{s_{t}\right\}$ exogenously-independently of debt
- MP sets $\left\{R_{t}\right\}$ to react weakly to inflation

$$
\begin{equation*}
\frac{\left(1+\rho P_{M t}\right) B_{M t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j} \tag{IEC}
\end{equation*}
$$

- In (IEC): right-side given, $B_{M t-1}$ predetermined
- (IEC) determines continuum of $\left(P_{t}, P_{M t}\right)$ combinations
- Can think of this as $E_{t} P V(s)$ determining

$$
\sum_{j=0}^{\infty}(\beta \rho)^{j} E_{t} \frac{P_{t-1}}{P_{t+j}}
$$

The expected present value of inflation

- Longer maturity-higher $\rho$-permits inflation to be postponed


## Alternative Consolidation \#1

- MP pegs $R_{t}=R^{*} \Rightarrow \pi_{t+j}=\pi^{*} \quad j \geq 1$
- all inflation occurs at $t$
- future inflation at $\pi^{*}=2 \%$
- $P_{M t}=P_{M}^{*}$ all $t \geq 0$
- (Not a realistic scenario, as it requires flexible prices)


## Alternative Consolidation \#1




Paths of Inflation \& Bond Prices: Debt-GDP from 80\% to 60\%

## Alternative Consolidation \#2

- Examine tradeoff between current \& (fixed) future inflation

$$
\begin{equation*}
\frac{\left(1+\rho P_{M t}\right) B_{M t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t} s_{t+j} \tag{IEC}
\end{equation*}
$$

- With fixed future inflation, $\pi^{F}$

$$
\begin{aligned}
P_{M t} & =\frac{\beta}{\pi^{F}-\rho \beta} \\
\frac{\pi^{F}}{\pi_{t}\left(\pi^{F}-\rho \beta\right)} & =\frac{E_{t} P V(s)}{b_{M t-1}}
\end{aligned}
$$

- Consolidation changes $E_{t} P V(s)$, given initial $b_{M t-1}$ at 80\%
- Note $\rho=0 \Rightarrow$ future inflation off the table


## Alternative Consolidations

|  | Current Inflation $\left(\pi_{t}\right)$ |  |  |
| :--- | ---: | ---: | ---: |
| Average | $2 \%$ |  | $4 \%$ |
| Maturity | $6 \%$ |  |  |
|  | Future Inflation $\left(\pi^{F}\right)$ |  |  |
| 5-year | 16.4 | 13.1 | 10.0 |
| 7-year | 10.8 | 8.0 | 5.4 |
| 10-year | 7.4 | 4.8 | 2.5 |
| 20-year | 3.9 | 1.7 | -0.4 |
| 30-year | 2.9 | 0.8 | -1.3 |
| 50-year | 2.1 | 0.1 | -2.0 |

Feasible Current Inflation ( $\pi_{t}$ ) and Future Inflation ( $\pi^{F}$ )
Combinations \& Average Debt Maturity (Annual \%)

## Alternative Consolidation \#2

- Longer average maturity, more can spread inflation over time
- Requires a particular monetary policy
- Long maturities imply small inflation cost to consolidation
- Some realities

1. in U.S., Fed has been shortening outstanding maturity via QE II \& III

- efforts to reduce long rates to stimulate growth

2. irony: with fears of deflation, this is precisely the policy to pursue
3. further irony: no policy makers are seriously considering this option

## Where To Go From Here

1. Employ new Keynesian model

- sticky prices: higher inflation lowers real interest rates
- lower real rates raise output, consumption, investment
- get an economic expansion from alternative consolidation

2. Introduce distorting taxes \& government spending
3. Compare welfare costs of conventional \& alternative consolidation
4. Brings back into the picture an old topic: optimal maturity structure of government debt

## Take Aways

- In a world where FP cannot be relied on to adjust surpluses as needed to stabilize debt. . .

1. it is impossible for MP to stabilize the economy
2. fiscal disturbances will always affect output, inflation \& interest rates
3. more aggressive MP will exacerbate the instability
4. fluctuations in "confidence" that affect real interest rates will transmit into fluctuations in output \& inflation
5. sudden flights to quality or away from junk can have real effects
6. tighter MP raises debt service, wealth, aggregate demand, and inflation

## Take Aways

1. Conventional perceptions of inflation miss a channel for fiscal inflation

- channel may be important in times of fiscal stress

2. Perception that MP can always stop an inflation that breaks out assumes the necessary fiscal backing will always be forthcoming

- when fiscal limit possible, the assumption breaks down

3. If inflation has fiscal roots, MP cannot offset it
4. Two policy options:
i. impose enforceable rules on fiscal behavior
ii. give different mandates to central banks

## A Monetary Union

- Two-country union (Sims, Bergin)
- world endowment: $Y_{t}=Y_{1, t}+Y_{2, t}=Y$
- household in country $j$ maximizes

$$
E_{0} \sum_{t=0}^{\infty} \beta^{t} u\left(C_{j, t}\right)
$$

subject to

$$
C_{j, t}+\frac{B_{j, t}}{P_{t}}+\tau_{j, t}=Y_{j, t}+z_{j, t}+\frac{R_{t-1} B_{j, t-1}}{P_{t}}
$$

- country j's government budget constraint

$$
\frac{D_{j, t}}{P_{t}}+\tau_{j, t}+v_{j, t}=z_{j, t}+\frac{R_{t-1} D_{j, t-1}}{P_{t}}
$$

$v_{j, t}$ : lump-sum transfers from central bank

- central bank's budget constraint

$$
\frac{B_{m, t}}{P_{t}}+v_{1, t}+v_{2, t}=\frac{R_{t-1} B_{m, t-1}}{P_{t}}
$$

## A Monetary Union

- Equilibrium conditions
- Euler equation for household $j$

$$
u^{\prime}\left(C_{j, t}\right)=\beta R_{t} E_{t} \frac{P_{t}}{P_{t+1}} u^{\prime}\left(C_{j, t+1}\right)
$$

- transversality condition for household $j$

$$
\lim _{T \rightarrow \infty} \beta^{T} E_{t} u^{\prime}\left(C_{j, t+T}\right) \frac{B_{j, t+T}}{P_{t+T}}=0
$$

- market clearing conditions

$$
\begin{aligned}
C_{1, t}+C_{2, t} & =Y_{1, t}+Y_{2, t}=Y \\
B_{1, t}+B_{2, t}+B_{m, t} & =D_{1, t}+D_{2, t}
\end{aligned}
$$

- Note: TVC applies to household's holdings of $B_{j, t}$, not to individual government issues, $D_{j, t}$
- can have eqm with $D_{1, t} \rightarrow+\infty$ and $D_{2, t} \rightarrow-\infty$


## A Monetary Union

- If $D_{1, t} \rightarrow+\infty$ and $D_{2, t} \rightarrow-\infty$, then govt 2 is completely financing govt 1 , with no expectation of repayment
- Not a stable political economy equilibrium
- Govt 2 can improve well-being of its citizens by refusing to do this
- Same argument applies to central bank
- We will impose individual govt and CB solvency

$$
\begin{aligned}
& \lim _{T \rightarrow \infty} \beta^{T} E_{t} u^{\prime}\left(C_{j, t+T}\right) \frac{D_{j, t+T}}{P_{t+T}}=0 \\
& \lim _{T \rightarrow \infty} \beta^{T} E_{t} u^{\prime}\left(C_{j, t+T}\right) \frac{B_{m, t+T}}{P_{t+T}}=0
\end{aligned}
$$

## A Monetary Union

- Assume $u\left(C_{j, t}\right)=C_{j, t}-\frac{a}{2} C_{j, t}^{2}$; adding Euler equations yields

$$
\frac{1}{R_{t}}=\beta E_{t} \frac{P_{t}}{P_{t+1}}
$$

- Applying this, country-specific consumptions are

$$
C_{1, t}=E_{t} C_{1, t+1}, \quad C_{2, t}=E_{t} C_{2, t+1}
$$

- Imposing eqm, get conditions

$$
\begin{aligned}
& \frac{R_{t-1} D_{1, t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[s_{1, t+j}+v_{1, t+j}\right] \\
& \frac{R_{t-1} D_{2, t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[s_{2, t+j}+v_{2, t+j}\right] \\
& \frac{R_{t-1} B_{m, t-1}}{P_{t}}=\sum_{j=0}^{\infty} \beta^{j} E_{t}\left[v_{1, t+j}+v_{2, t+j}\right]
\end{aligned}
$$

## A Monetary Union

- Policy assumptions
- CB pegs nominal rate: $R_{t}=R^{*}$
- country 1 raises surpluses passively with debt
- country 2 sets surpluses independent of debt
- CB rebates portfolio earnings to countries, independent of their debt
- Results

1. Union-wide inflation determined by country 2 (one with profligate FP)
2. News about country 2 surpluses affects inflation \& value of debt in both countries
3. Requires adjustments in country 1's surpluses

## A Monetary Union

- How can CB retain control of inflation?
- rebates to countries depend on each nation's debt in the right way
- make MP active (ECB in normal times)
- Efforts by the CB to reduce inflation
- raise value of debt in both countries
- requires higher rebates from CB to country 2 (backs debt of profligate country)
- rebates to country 1 may need to be negative (taxes)
- gives CB power to tax and transfer
- Message: A fiscal union can support monetary union's efforts to control inflation


## A Monetary Union: Needed Extensions

- This model has a single union-wide price level
- Need a model that can account for persistent differences in inflation rates across member nations
- Need heterogeneity across member nations
- Need size of economy to matter
- These are open modeling issues of great importance in Eurozone
- How are European inflation rates determined?


## Generalizing the Model

- Up to now, have focused on endowment economies
- exogenous real interest rate convenient for analytics
- but can be misleading
- Now introduce nominal rigidities
- track MP \& FP impacts on real rates and output
- Employ textbook new Keynesian model
- Focus on Regime F since Regime M well covered by textbooks


## Nominal Rigidities

- Follows Woodford (1998)
- Sticky prices: fraction $1-\alpha$ of goods suppliers get to set a new price each period
- Continuum of identical households indexed by $j \in[0,1]$, each specializes in production of single differentiated good
- Continuum of differentiated goods each period indexed by $z \in[0,1]$
- Household $j$ maximizes

$$
E_{0}\left\{\sum_{j=0}^{\infty} \beta^{t}\left[u\left(C_{t}^{j}\right)+v\left(\frac{M_{t}^{j}}{P_{t}}\right)-w\left(y_{t}(j)\right)\right]\right\}
$$

where $y_{t}(j)$ : HH j's supply of its product and

$$
C_{t}^{j} \equiv\left[\int_{0}^{1} c_{t}^{j}(z)^{\frac{\theta-1}{\theta}} d z\right]^{\frac{\theta}{\theta-1}}, \quad \theta>1
$$

## Nominal Rigidities

- Household j's budget constraint

$$
\int_{0}^{1} p_{t}(z) c_{t}^{j}(z) d z+M_{t}^{j}+Q_{t, t+1} B_{t}^{j} \leq W_{t}^{j}+p_{t}(j) y_{t}(j)-P_{t} \tau_{t}
$$

with $P_{t} \equiv\left[\int_{0}^{1} p_{t}(z)^{1-\theta} d z\right]^{\frac{1}{1-\theta}}$ and $W_{t}^{j} \equiv M_{t-1}+B_{t-1}^{j}$

- Government's budget constraint

$$
Q_{t, t+1} B_{t}=B_{t-1}+P_{t} \Delta_{t}-\left(M_{t}-M_{t-1}\right)
$$

with $\Delta_{t} \equiv-\tau_{t}$, primary deficit

- Aggregate resource constraint: $C_{t}=Y_{t}$


## Nominal Rigidities

- Equilibrium conditions

$$
\begin{aligned}
& Q_{t, T}=\beta^{T-t} \frac{u^{\prime}\left(Y_{T}\right)}{u^{\prime}\left(Y_{t}\right)} \frac{P_{t}}{P_{T}} \\
& \frac{v^{\prime}\left(M_{t} / P_{t}\right)}{u^{\prime}\left(Y_{t}\right)}=\frac{R_{t}-1}{R_{t}} \\
& \frac{1}{R_{t}}=\beta E_{t}\left[\frac{u^{\prime}\left(Y_{t+1}\right)}{u^{\prime}\left(Y_{t}\right)} \frac{P_{t}}{P_{t+1}}\right] \\
& \lim _{T \rightarrow \infty} E_{t}\left[Q_{t, T} W_{T}\right]=0
\end{aligned}
$$

- Integrating over all households, intertemporal HH bc

$$
\begin{aligned}
\sum_{T=t}^{\infty} & E_{t}\left\{Q_{t, T}\left[P_{T} C_{T}+\frac{R_{T}-1}{R_{T}} M_{T}\right]\right\} \\
& =\sum_{T=t}^{\infty} E_{t}\left\{Q_{t, T}\left[P_{T} Y_{T}-P_{T} \tau_{T}\right]\right\}+M_{t-1}+B_{t-1}
\end{aligned}
$$

## Nominal Rigidities

- Price-setting behavior
- HH chooses new price, $P_{t}^{*}$, to satisfy

$$
\sum_{k=0}^{\infty} \alpha^{k} E_{t}\left\{Q_{t, t+k} Y_{t+k}\left(\frac{P_{t}^{*}}{P_{t+k}}\right)^{-\theta}\left[P_{t}^{*}-\mu S_{t+k, t}\right]\right\}=0
$$

where $\mu \equiv \theta /(\theta-1)>1$ : markup

- $S_{T, t}$ : marginal cost at $T$ of good whose price was set at $t$

$$
S_{T, t}=\frac{w^{\prime}\left(Y_{T}\left(\frac{P_{t}^{*}}{P_{T}}\right)^{-\theta}\right)}{u^{\prime}\left(Y_{T}\right)} P_{T}
$$

- and price index is

$$
P_{t}=\left[\alpha P_{t-1}^{1-\theta}+(1-\alpha) P_{t}^{*(1-\theta)}\right]^{\frac{1}{1-\theta}}
$$

- Flexible prices: $P_{t}^{*}=\mu S_{t, t}$, so $P_{t}=P^{*}, Y_{t}=Y^{*}$ where $Y^{*}$ solves $u^{\prime}\left(Y^{*}\right)=\mu w^{\prime}\left(Y^{*}\right)$


## Fiscal Policy as Source of Instability

- Suppose there are no constraints on FP, so $\left\{\Delta_{t}\right\}$ is exogenous
- Then fiscal disturbances must affect inflation, output, and interest rates, regardless of MP behavior
- Proof by Contradiction: Suppose there is a MP that delivers stable prices despite fluctuations in $\Delta_{t}$
- then $Y_{t}=Y^{*}$ all $t$
- $R_{t}$ and $M_{t}$ constant and

$$
\begin{aligned}
Q_{t, T}=\beta^{T-t}, \quad R^{*} & =\beta^{-1}, \quad C_{t}=Y^{*} \\
\sum_{j=0}^{\infty} \beta^{j} \frac{R^{*}-1}{R^{*}} m^{*} & =m^{*}
\end{aligned}
$$

- HH's intertemporal budget constraint is

$$
\frac{W_{t}}{P^{*}}=m^{*}-\delta_{t}
$$

where $\delta_{t} \equiv \sum_{j=0}^{\infty} \beta^{j} E_{t} \Delta_{t+j}$

## Fiscal Policy as Source of Instability

$$
\begin{align*}
\frac{W_{t}}{P^{*}} & =m^{*}-\delta_{t}  \tag{IBC}\\
\delta_{t} & \equiv \sum_{j=0}^{\infty} \beta^{j} E_{t} \Delta_{t+j}
\end{align*}
$$

- But $W_{t}$ predetermined at $t$
- Equilibrium condition (IBC) $\Rightarrow$ fiscal shock cannot change $\delta_{t}$
- Conclusion: Random variation in FP necessarily inconsistent with price stability
- Conclusion is independent of MP behavior
- so nothing MP can do to offset instability


## Analytics for Cashless Limit Version

- Four-equation system

$$
\begin{aligned}
y_{t} & =E_{t} y_{t+1}-\sigma\left(i_{t}-E_{t} \pi_{t+1}\right) \\
\pi_{t} & =\beta E_{t} \pi_{t+1}+\kappa y_{t} \\
b_{t} & =i_{t}+\beta^{-1}\left(b_{t-1}-\pi_{t}\right)+\left(\beta^{-1}-1\right) \Delta_{t} \\
i_{t} & =\alpha \pi_{t}+\varphi_{t}
\end{aligned}
$$

- Can show that

$$
(1-\alpha \beta) \sum_{j=0}^{\infty} \beta^{j} E_{t} \pi_{t+j}=b_{t-1}+\beta \sum_{j=0}^{\infty} \beta^{j} E_{t} \varphi_{t+j}+(1-\beta) \sum_{j=0}^{\infty} \beta^{j} E_{t} \Delta_{t+j}
$$

1. present value of inflation determined by policy shocks
2. more hawkish MP—higher $\alpha$ —amplifies positive impacts of deficits \& interest rates

## Analytics for Cashless Limit Version

- Flexible-price case: $\kappa=\infty \Rightarrow y_{t} \equiv 0$
- Constant real rate: $i_{t}=E_{t} \pi_{t+1}$
- Note that

$$
E_{t} \pi_{t+j}=\alpha^{j} \pi_{t}+\alpha^{j-1} \varphi_{t}+\alpha^{j-2} E_{t} \varphi_{t+1}+\ldots+\alpha E_{t} \varphi_{t+j-2}+E_{t} \varphi_{t+j-1}
$$

- Solve for $\pi_{t}$ from ( $\dagger$ )

$$
\pi_{t}=b_{t-1}+\beta(1-\alpha \beta) \sum_{j=0}^{\infty} \beta^{j} E_{t} \varphi_{t+j}+(1-\beta) \sum_{j=0}^{\infty} \beta^{j} E_{t} \Delta_{t+j}
$$

1. higher inflation from higher PV deficits or interest rates
2. effect of deficits on $\pi_{t}$ not affected by MP
3. more hawkish MP increases effect of deficits on expected $\pi$

- Note: $E_{t} \pi_{t+1}$ from ( $\ddagger$ ) consistent


## Analytics for Cashless Limit Version

- Return to sticky-price model: $0<\kappa<\infty$
- output and real interest rate endogenous
- Real rate: $r_{t+j} \equiv i_{t+j-1}-\pi_{t+j}$
- Rewrite ( $\dagger$ ) as

$$
\pi_{t}-\sum_{j=1}^{\infty} \beta^{j} E_{t} r_{t+j}=b_{t-1}+(1-\beta) \sum_{j=0}^{\infty} \beta^{j} E_{t} \Delta_{t+j}
$$

- News about higher deficits shows up as a mix of

1. higher current inflation
2. lower path of real interest rates
3. transmits to higher output
4. MP behavior determines split between inflation \& real activity

## Analytics for Cashless Limit Version

- Combine Euler equation, Phillips curve, MP rule

$$
E_{t} \pi_{t+2}-\beta^{-1}(1+\beta+\sigma \kappa) E_{t} \pi_{t+1}+\beta^{-1}(1+\alpha \sigma \kappa) \pi_{t}=-\beta^{-1} \sigma \kappa \varphi_{t}
$$

- Can show two real roots: $\left|\lambda_{1}\right|<1,\left|\lambda_{2}\right|>1$
- Solution for expected inflation

$$
E_{t} \pi_{t+1}=\lambda_{1} \pi_{t}+\left(\beta \lambda_{2}\right)^{-1} \sigma \kappa \sum_{j=0}^{\infty} \lambda_{2}^{-j} E_{t} \varphi_{t+j}
$$

- Solve recursively given exogenous $\left\{\Delta_{t}, \varphi_{t}\right\}$, predetermined $b_{t-1}$

1. solve for $\pi_{t}$ from ( $\dagger$ )
2. $\pi_{t} \& E_{t} \pi_{t+1}$ yield $y_{t}$
3. $i_{t}$ from MP rule
4. $b_{t}$ from government budget constraint
5. repeat

## Return to Cash Version with Exogenous FP

- Assume MP rule that doesn't react to fiscal variables

$$
R_{t}=\Phi\left(\pi_{t}, Y_{t}\right)
$$

- Government issues only 1-period nominal debt

$$
B_{t}=R_{t}\left[B_{t-1}+P_{t} \Delta_{t}-\left(M_{t}-M_{t-1}\right)\right]
$$

- Steady state is

$$
\Delta_{t}=\Delta^{*}<0, \quad \Phi\left(1, Y^{*}\right)=\beta^{-1}-1, \quad R^{*}=\beta^{-1}
$$

- Log-linearize system around steady state


## Equilibrium Consistent with Exogenous FP

- System is $\left(\hat{x}_{t} \equiv \ln \left(x_{t}\right)-\ln \left(x^{*}\right)\right)$

$$
\begin{aligned}
\hat{m}_{t} & =\chi\left[\sigma^{-1} \hat{Y}_{t}-\left(\frac{\beta}{1-\beta}\right) \hat{R}_{t}\right] \\
\hat{Y}_{t} & =E_{t} \hat{Y}_{t+1}-\sigma\left(\hat{R}_{t}-E_{t} \hat{\pi}_{t+1}\right) \\
\hat{R}_{t} & =\phi_{\pi} \hat{\pi}_{t}+\phi_{Y} \hat{Y}_{t} \\
\hat{b}_{t} & =\hat{R}_{t}+\beta^{-1}\left(\hat{b}_{t-1}+\hat{\pi}_{t}\right)+\left(\beta^{-1}-1\right) \hat{\Delta}_{t}+\gamma\left(\hat{m}_{t-1}-\hat{m}_{t}-\hat{\pi}_{t}\right) \\
\hat{\pi}_{t} & =\beta E_{t} \hat{\pi}_{t+1}+\kappa \hat{Y}_{t}
\end{aligned}
$$

$$
\text { where } \hat{\Delta}_{t} \equiv \frac{\Delta^{*}-\Delta_{t}}{\Delta^{*}}, \sigma \equiv-\frac{u^{\prime}\left(Y^{*}\right)}{u^{\prime \prime}\left(Y^{*}\right) Y^{*}}, \chi \equiv \frac{v^{\prime}\left(m^{*}\right)}{v^{\prime \prime}\left(m^{*}\right) m^{*}}, \gamma \equiv \frac{m^{*}}{\beta b^{*}}
$$

$$
\kappa \equiv \frac{(1-\alpha)(1-\alpha \beta)}{\alpha} \frac{\omega+\sigma}{\sigma(\omega+\theta)}, \omega \equiv \frac{\omega^{\prime}\left(Y^{*}\right)}{w^{\prime \prime}\left(Y^{*}\right) Y^{*}}
$$

- Solve for $\left\{\hat{Y}_{t}, \hat{\pi}_{t}, \hat{R}_{t}, \hat{b}_{t}, \hat{m}_{t}\right\}$ given $\hat{\Delta}_{t}=\rho \hat{\Delta}_{t-1}+\varepsilon_{t}$


## Impacts of Deficit

- With $\left\{\hat{\Delta}_{t}\right\}$ exogenous, unique eqm requires relatively weak reactions to inflation and output

$$
-1-\frac{1+\beta}{\kappa} \phi_{Y}-\frac{2(1+\beta)}{\kappa \sigma}<\phi_{\pi}<1-\frac{1-\beta}{\kappa} \phi_{Y}
$$

- Benchmark calibration

$$
\beta=.95, \kappa=.3, \chi=\sigma=1, \gamma=.1, \rho=.6, Y^{*}=1, b^{*} / Y^{*}=.5
$$

- Vary MP choices of $\phi_{\pi}$ and $\phi_{Y}$

Pegged interest rate: $\phi_{\pi}=\phi_{Y}=0$
Weak lean against wind: $\phi_{\pi}=\phi_{Y}=.3$
Aggressive stance: $\phi_{\pi}=.9, \phi_{Y}=.5$

## Impacts of Deficit: Pegged Rate




Nominal Rate



## Impacts of Deficit: Pegged Rate






## Impacts of Deficit: More Hawkish

Output


Nominal Rate




## Impacts of Deficit: More Hawkish






## Impacts of Deficit: Even More Hawkish






## Impacts of Deficit: Even More Hawkish






## Sources of Fiscal Financing

- Write government budget constraint as

$$
\begin{aligned}
\hat{b}_{t}+E_{t} \hat{\delta}_{t+1} & =\hat{R}_{t}+\beta^{-1}\left(\hat{b}_{t-1}+\hat{\delta}_{t}-\hat{\pi}_{t}\right)+\gamma\left(\hat{m}_{t-1}-\hat{m}_{t}-\hat{\pi}_{t}\right) \\
\hat{\delta}_{t} & \equiv(1-\beta) \sum_{j=0}^{\infty} \beta^{j} E_{t} \hat{\Delta}_{t+j}
\end{aligned}
$$

- Solving for the present value of deficits

$$
\begin{aligned}
& \hat{\delta}_{t}=\underbrace{-\left(\hat{b}_{t-1}-\hat{\pi}_{t}\right)}_{\text {surprise revaluation }}+\gamma \sum_{\mathrm{PV}(\text { seigniorage })}^{\gamma \sum_{j=0}^{\infty} \beta^{j+1} E_{t} \hat{\mu}_{t+j}} \underbrace{-\sum_{j=0}^{\infty} \beta^{j+1} E_{t}\left[\hat{R}_{t+j}-\hat{\pi}_{t+j+1}\right]}_{\mathrm{PV}(\text { real discount rates })} \\
& \hat{\mu}_{t} \equiv \hat{m}_{t}-\hat{m}_{t-1}+\hat{\pi}_{t}
\end{aligned}
$$

## Quantitative Implications

|  | Percentage Due to |  |  | \% Change in |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\hat{\pi}_{t}$ | PV (seig) | $-\mathrm{PV}(\mathrm{r})$ | $\mathrm{PV}(\pi)$ | $\mathrm{PV}(\mathrm{Y})$ |
| $\begin{gathered} \gamma=.1 \\ \phi_{\pi}=\phi_{Y}=0 \end{gathered}$ | 39.6 | 9.4 | 51.0 | 2.1 | 3.1 |
| $\phi_{\pi}=\phi_{Y}=.3$ | 52.5 | 9.4 | 38.1 | 4.6 | 4.1 |
| $\begin{gathered} \phi_{\pi}=.9, \phi_{Y}=.5 \\ \gamma=0 \end{gathered}$ | 88.4 | 10.0 | 1.6 | 36.9 | 6.9 |
| $\phi_{\pi}=\phi_{Y}=0$ | 43.7 | 0 | 56.3 | 2.3 | 3.4 |
| $\phi_{\pi}=.9, \phi_{Y}=.5$ | 98.1 | 0 | 1.9 | 41.0 | 7.6 |

Dynamic Impacts of Exogenous Serially Correlated Deficit Increase seig: seigniorage; $r$ : real discount rate; $\mathrm{PV}(\mathrm{X})$ : present-value change in X; $\gamma \equiv m^{*} /\left(\beta b^{*}\right) ; \phi_{\pi}, \phi_{Y}$ : MP parameters

## Implications: Monetary Policy Effects

- An open-market sale of $B$ reduces $M$, raises $R$
- If higher nominal $R$ means higher real $r$
- holding FP fixed, this lowers $E_{t} P V(s)$
- induces people to substitute out of government debt, into goods
- raises aggregate demand
- highly irregular
- Conventional view implicitly requires FP to generate higher expected surpluses
- If surpluses rise enough to raise $E_{t} P V(s)$, even with higher real discount rates...
- tighter MP reduces demand and inflation
- otherwise, demand and inflation rise


## Implications: Monetary Policy Effects

- In new Keynesian model

$$
\begin{aligned}
& \hat{Y}_{t}=E_{t} \hat{Y}_{t+1}-\sigma\left(\hat{R}_{t}-E_{t} \hat{\pi}_{t+1}\right) \\
& \hat{\pi}_{t}=\beta E_{t} \hat{\pi}_{t+1}+\kappa \hat{Y}_{t}
\end{aligned}
$$

$$
\hat{\pi}_{t}=\underbrace{\left(\hat{b}_{t-1}-\hat{\delta}_{t}\right)}_{=0} \underbrace{-\gamma \sum_{j=0}^{\infty} \beta^{j+1} E_{t} \hat{\mu}_{t+j}}_{\mathrm{PV}(\text { seigniorage })} \underbrace{\sum_{j=0}^{\infty} \beta^{j+1} E_{t}\left[\hat{R}_{t+j}-\hat{\pi}_{t+j+1}\right]}_{\mathrm{PV}(\text { real discount rates })}
$$

- Tighter monetary policy with fixed surpluses
- raises $\hat{R}_{t}-E_{t} \hat{\pi}_{t+1}$ in short run: lowers output
- raises entire path of $\left\{E_{t} \hat{\pi}_{t+j}\right\}$ : raise inflation
- appears as an adverse shift in the Phillips curve
- More hawkish MP—stronger response to inflation—prolongs rise in $r$
- higher real debt service enhances wealth effects
- raises inflation still more


## Implications: Monetary Policy Effects








Serially correlated exogenous monetary policy contraction

## Implications: Monetary Policy Effects








Serially correlated exogenous monetary policy contraction

## Real Discount Rates

$$
\frac{M_{t-1}+Q_{t} B_{t-1}}{P_{t}}=E_{t} \sum_{j=0}^{\infty} \frac{1}{r_{t, t+j}} s_{t+j}
$$

- $r_{t, t+j}$ is $j$-step-ahead real discount rate
- Adjustments to eqm need not occur through $s_{t+j}$
- price rigidities make future $r$ 's important source of financing
- Changes in $E_{t} P V(s)$ need not occur through $s_{t+j}$
- variations in expected $r$ 's can have big effects on $E_{t} P V(s)$ with no change in $s$ 's
- Leads to dramatic re-interpretations


## Flight to Quality

$$
\frac{M_{t-1}+Q_{t} B_{t-1}}{P_{t}}=E_{t} \sum_{j=0}^{\infty} \frac{1}{r_{t, t+j}} s_{t+j}
$$

- Flight to quality in financial crises and recessions
- Investors hold debt at lower expected returns
- As demand for debt rises, demand for goods falls
- Lower demand reduces inflation
- Intertemporal equilibrium condition's role
- lower $r$ 's raises $E_{t} P V(s)$ if surpluses unresponsive
- higher $E_{t} P V(s)$ raises value of debt
- Fluctuating discount rates can be a source of business cycles in Regime F-not in Regime M
- MP response: raise rates to increase aggregate demand


## Implications: Discount Rates

- The 2008-2009 recession: conventional story doesn't hold up (Cochrane)
- Sharp increase in precautionary demand for money
- not met by supply
- $\Rightarrow$ lower demand \& real output
- Fed flooded economy with reserves
- no flight to money, out of bonds
- no bank runs
- Instead, a flight to all quality- $M$ \& $B$-out of goods
- Similar to convention, but focuses on all government debt, rather than just money
- Appropriate policy responses?
- announce cuts in fiscal surpluses
- if surpluses fixed and MP can affect real interest rates, then MP should raise rates
- Highly irregular


## Exchange-Rate \& Price-Level Determination

- Merge intertemporal approaches to current account \& fiscal policy to determine price level \& nominal exchange rate
- Combine two intertemporal equilibrium conditions to create a consolidated present-value condition relevant for open economies
- Simultaneously determine
- equilibrium price level $(P)$
- equilibrium nominal exchange rate (e)
- Price level \& exchange rate reflect interactions between expected present values of budget surpluses \& trade balances $(E P V(S) \& E P V(N X))$
- generalizes fiscal theory of price level
- Draws on work-in-progress with Tack Yun


## World Economic Developments

- Recent years have seen in advanced economies
- persistently high levels of public debt
- large holdings by some governments of international reserves
- prolonged period of passive-zero lower bound-monetary policies
- Provide a rationale for accumulation of international reserves to achieve price-level \& exchange-rate targets
- more reserves $\Rightarrow$ lower price level
- because government has more assets to support outstanding public debt
- If reserves exceed public debt, fiscal policy can be sustainable
- even if expected present value of surpluses is negative


## Overview

- Derive these implications using a general economic structure
- budget constraints
- asset-pricing relations
- market-clearing conditions
- transversality \& other limiting conditions
- economy at cashless limit
- Because we do not specify complete model, we are deriving equilibrium relationships
- may or may not have causal interpretations
- in spirit of "intertemporal approach to current account" literature


## Overview

- Introduce composition effects from asset demands
- $\alpha_{t}$ : share of foreign assets held by households (rest held by government)
- $\gamma_{t}$ : fraction of government debt held by financial sector (rest held by households)
- for now, take these asset demands as given (unmodeled)
- $\alpha_{t} \equiv 1$ (govt owns no foreign assets) $\& \gamma_{t} \equiv 0$ (all govt debt held by HHs)
- obtain usual bond-valuation equation in fiscal theory
- no role for trade balances in price determination


## Overview

- $\alpha_{t}<1$ (govt owns foreign assets)
- $\alpha_{t}$ can be negative
- trade balances \& budget surpluses affect $P$ \& $e$ determination
- composition effects matter
- alters how to think about fiscal sustainability... depends on
- govt holdings of foreign assets
- $E P V(S) \& E P V(N X)$
- relative asset demands
- How important it is to integrate intertemporal approaches to CA \& FP...
- depends on country characteristics
- asset demands
- relative sizes of $E P V(S) \& E P V(N X)$


## Benchmark Model

- Representative household's budget constraint

$$
\begin{equation*}
\frac{q_{t} B_{t+1}^{H}+e_{t} q_{t}^{*} B_{t+1}^{*}}{P_{t}}=\frac{B_{t}^{H}+e_{t} B_{t}^{*}}{P_{t}}+W_{t} H_{t}+R_{t} K_{t}-C_{t}-T_{t}-I_{t} \tag{1}
\end{equation*}
$$

$B_{t+1}^{H}$ : nominal $¥$ value of government bonds issued by home country
$B_{t+1}^{*}$ : nominal $\$$ value of government bonds issued by foreign country $e_{t}$ : nominal exchange rate in $¥ / \$$
$q_{t}: ¥$ price of home country bonds
$q_{t}^{*}: \$$ price of foreign country bonds
$W_{t}$ : real wage rate; $R_{t}$ : real rental rate of capital
$H_{t}$ : number of hours worked; $K_{t}$ : capital stock at beginning of $t$
$C_{t}$ : consumption; $I_{t}$ : investment; $T_{t}$ : real taxes

- Note that $B_{t}$ is bonds outstanding at the beginning of period $t$


## Model Setup

## Assumption (1)

There is an international financial market in which risk-neutral international investors trade bonds issued by different governments. In this market, uncovered interest parity holds

$$
\begin{equation*}
q_{t}^{*} e_{t}=q_{t} E_{t}\left[e_{t+1}\right] \tag{2}
\end{equation*}
$$

Assumption (2)
Markets for factors of production and goods are perfectly competitive. Production exhibits constant returns to scale, so

$$
\begin{equation*}
Y_{t}=W_{t} H_{t}+R_{t} K_{t} \tag{3}
\end{equation*}
$$

$Y_{t}$ is aggregate output in period $t$

## Model Setup

- Aggregate resource constraint is

$$
\begin{equation*}
Y_{t}=C_{t}+I_{t}+G_{t}+N X_{t} \tag{4}
\end{equation*}
$$

$G_{t}$ is real government consumption; $N X_{t}$ is real net exports

- Substitute (2) \& (4) into (1) to yield difference equation in household's real asset holdings, $d_{t}$

$$
\begin{equation*}
E_{t}\left[q_{t, t+1} d_{t+1}\right]=d_{t}+G_{t}-T_{t}+N X_{t} \tag{5}
\end{equation*}
$$

where

$$
d_{t} \equiv \frac{B_{t}^{H}+e_{t} B_{t}^{*}}{P_{t}}, \quad q_{t, t+1} \equiv q_{t} \frac{P_{t+1}}{P_{t}}
$$

- Use one-period real discount rate, $q_{t, t+1}$ to define

$$
q_{t, t+k}=\prod_{j=1}^{k} q_{t+j-1, t+j}, \quad k \geq 1, \quad q_{t, t}=1
$$

## Model Setup

- Iterate forward on (5) \& impose transversality $\lim _{T \rightarrow \infty} E_{t}\left[q_{t, t+T} d_{t+T}\right]=0$ to get

$$
\begin{equation*}
\frac{B_{t}^{H}+e_{t} B_{t}^{*}}{P_{t}}=\sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k}\left(T_{t+k}-G_{t+k}-N X_{t+k}\right)\right] \tag{6}
\end{equation*}
$$

- assume $\operatorname{EPV}(S)>E P V(N X)$
- Budget $\&$ trade surpluses back $B_{t}^{H}+e_{t} B_{t}^{*}$
- household's total holdings of nominal govt bonds
- If $E P V(S) \uparrow$ or $E P V(N X) \downarrow \ldots$
- expected higher "cash flows"
- demand for $B_{t}^{H}+e_{t} B_{t}^{*}$ rises
- mix of $P_{t} \downarrow \& e_{t} \uparrow$
- From only the HH's side, $P_{t} \& e_{t}$ move in opposite directions


## Model Setup

## Assumption (3)

The government holds dollar-denominated foreign assets, $J_{t}$, at the beginning of each $t \geq 0 . J_{t}$ is determined in period $t-1$ and assume that the government does not adjust the exchange rate in response to changes in $J_{t}$

- Government's flow budget constraint is

$$
\begin{equation*}
q_{t} B_{t+1}^{G}-e_{t} q_{t}^{*} J_{t+1}+P_{t} T_{t}=B_{t}^{G}-e_{t} J_{t}+P_{t} G_{t} \tag{7}
\end{equation*}
$$

$B_{t}^{G}$ is $¥$ value of outstanding government debt at beginning of $t ; J_{t+1}$ is the $\$$ value of foreign assets held by home country; $e_{t} J_{t+1}$ is $¥$ value of foreign assets held by govt

## Model Setup

- Applying uncovered interest parity, (7) is

$$
\begin{equation*}
E_{t}\left[q_{t, t+1} b_{t+1}\right]=b_{t}-\left(T_{t}-G_{t}\right) \tag{8}
\end{equation*}
$$

where $b_{t} \equiv \frac{B_{t}^{G}-e_{t} J_{t}}{P_{t}}$

## Assumption (4)

Total outstanding public debt-net of the government's holdings-consists of the household's holdings, $B_{t}^{H}$, and the financial sector's holdings, $B_{t}^{F}$

$$
B_{t}^{G}=B_{t}^{H}+B_{t}^{F}
$$

- We do not model how $B_{t}^{F}$ is determined
- think of it as foreign lending to domestic govt


## Model Setup

- Iterate forward on (8) and impose limiting condition $\lim _{T \rightarrow \infty} E_{t}\left[q_{t, t+T} b_{t+T}\right]=0$ to yield

$$
\begin{equation*}
\frac{B_{t}^{G}-e_{t} J_{t}}{P_{t}}=\sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k}\left(T_{t+k}-G_{t+k}\right)\right] \tag{9}
\end{equation*}
$$

- Budget surpluses alone back $B_{t}^{G}-e_{t} J_{t}$
- government's net nominal liabilities
- Two interpretations of (9)

1. $B_{t}^{G}-e_{t} J_{t}$ : "net debt" of govt backed by $E P V(S)$
2. $B_{t}^{G} / P_{t}=E P V(S)+\left(e_{t} / P_{t}\right) J_{t}$

- $\left(e_{t} / P_{t}\right) J_{t}$ : income source at $t$ (capital inflows) that supplement $\operatorname{EPV}(S)$ as backing for govt bonds
- By (9), $E P V(S) \uparrow \Rightarrow \operatorname{mix}$ of $P_{t} \downarrow \& e_{t} \downarrow$
- From only the govt's side, $P_{t} \& e_{t}$ move in same direction


## Model Setup

- Can instead combine HH's and govt's budget constraints, impose marketing clearing

$$
\frac{e_{t} q_{t}^{*}\left(J_{t+1}+B_{t+1}^{*}\right)-q_{t} B_{t+1}^{F}}{P_{t}}=\frac{e_{t}\left(J_{t}+B_{t}^{*}\right)-B_{t}^{F}}{P_{t}}+N X_{t}
$$

- applying uncovered interest parity

$$
E_{t}\left[q_{t, t+1} f_{t+1}\right]=f_{t}+N X_{t}
$$

where $f_{t} \equiv \frac{e_{t}\left(J_{t}+B_{t}^{*}\right)-B_{t}^{F}}{P_{t}}$

- iterate forward \& impose $\lim _{T \rightarrow \infty} E_{t}\left[q_{t, t+T} f_{t+T}\right]=0$

$$
\begin{equation*}
\frac{e_{t}\left(J_{t}+B_{t}^{*}\right)-B_{t}^{F}}{P_{t}}=\sum_{k=0}^{\infty} E_{t} q_{t, t+k} N X_{t+k} \tag{10}
\end{equation*}
$$

- (10) equates the value of net lending to foreigners to EPV $(N X)$
- conventional intertemporal approach to current account expression


## Equilibrium

- Two intertemporal equilibrium conditions-(6) \& (9)imply different movements between $P_{t} \& e_{t}$
- Need to bring both conditions together
- (6): value of HH asset holdings equals expected present value of govt surpluses net of trade surpluses
- (9): value of net govt liabilities equals expected present value of primary govt surpluses
- in closed economy ( $\left.N X_{t}=B_{t}^{*}=J_{t} \equiv 0\right)$, these are identical
- in open economy, both must hold simultaneously, imposing additional restrictions on equilibrium price level


## Price-Level Determination

- Combining (6) \& (9) to eliminate $e_{t}$ yields

$$
\begin{aligned}
\frac{B_{t}^{H}+\alpha_{t} B_{t}^{F}}{P_{t}} & =\sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k}\left(T_{t+k}-G_{t+k}\right)\right] \\
& -\left(1-\alpha_{t}\right) \sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k} N X_{t+k}\right]
\end{aligned}{ }_{\alpha_{t} \equiv \frac{B_{t}^{*}}{B_{t}^{*}+J_{t}}=\left\{\begin{array}{l}
\text { share of foreign assets } \\
\text { held by households }
\end{array}\right.} .
$$

- Price-level determination: Two results when assumptions 1-4 hold:

1. If govt holds foreign assets, equilibrium price level reflects both primary budget surpluses \& trade surpluses
2. When the financial sector holds govt bonds, there are composition effects of debt on the price level

## Price-Level Determination

- Denote by $\gamma_{t}$ the fraction of govt debt held by financial sector

$$
\gamma_{t} \equiv B_{t}^{F} / B_{t}^{G}
$$

- Solving for the price level from (11) yields

$$
\begin{equation*}
P_{t}=\frac{\left[\left(1-\gamma_{t}\right)+\gamma_{t} \alpha_{t}\right] B_{t}^{G}}{\sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k}\left(T_{t+k}-G_{t+k}\right)\right]-\left(1-\alpha_{t}\right) \sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k} N X_{t+k}\right]} \tag{12}
\end{equation*}
$$

- Treat (12) as determining the equilibrium price level at $t$
- requires assuming $\left\{T_{t}-G_{t}, N X_{t}\right\}$ unrelated to govt debt \& price level (e.g., exogenous)
- at $t, B_{t}^{G}, \alpha_{t}, \gamma_{t}$ predetermined
- with $B_{t}^{G}>0$, also assume that for all $t \geq 0$


## Price-Level Determination

$$
\begin{equation*}
P_{t}=\frac{\left[\left(1-\gamma_{t}\right)+\gamma_{t} \alpha_{t}\right] B_{t}^{G}}{E_{t} P V(S)-\left(1-\alpha_{t}\right) E_{t} P V(N X)} \tag{12}
\end{equation*}
$$

$$
E P V(S)>E P V(N X) \Rightarrow E P V(S)>(1-\alpha) E P V(N X)
$$

1. $\alpha_{t} \& \gamma_{t}$ yield composition effects

- both $\alpha_{t} \& \gamma_{t}$ are predetermined
- $\alpha_{t}=1, \gamma_{t}=0$ yields fiscal theory $P$ determination

2. When $0<\alpha_{t}<1 \ldots$

- higher $E P V(N X)$ raises $P_{t}$

3. When $0<\gamma_{t}<1$, higher $\gamma_{t} \ldots$

- raises $P_{t}$ because $E P V(S)>(1-\alpha) E P V(N X)$

4. Higher $\alpha_{t} \ldots$

- raises $P_{t}$ if $\gamma E P V(S)>E P V(N X)$
- lowers $P_{t}$ if $\gamma E P V(S)<E P V(N X)$

5. When $\gamma_{t}=0$, higher $\alpha_{t}$ decreases $P_{t}$ as long as $E P V(N X)>0$

## Exchange-Rate Determination

- Exchange-rate determination: Two results when assumptions 1-4 hold:

1. If the govt holds foreign assets $\left(J_{t}>0\right)$, nominal exchange rate reflects both primary budget surpluses \& trade surpluses
2. When the financial sector holds govt bonds $\left(B_{t}^{F}>0\right)$, there are composition effects on of debt on the exchange rate

- Substitute (12) into (9) \& solve for exchange rate

$$
\begin{equation*}
e_{t}=\left(1-\alpha_{t}\right) \frac{B_{t}^{G}}{J_{t}} \frac{\gamma_{t} \sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k}\left(T_{t+k}-G_{t+k}\right)\right]-\sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k} N X_{t+k}\right]}{\sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k}\left(T_{t+k}-G_{t+k}\right)\right]-\left(1-\alpha_{t}\right) \sum_{k=0}^{\infty} E_{t}\left[q_{t, t+k} N X_{t+k}\right]} \tag{13}
\end{equation*}
$$

- Treat (13) as determining the equilibrium exchange rate
- requires same assumptions as for price-level


## Exchange-Rate Determination

$$
\begin{equation*}
e_{t}=\left(1-\alpha_{t}\right) \frac{B_{t}^{G}}{J_{t}}\left[\frac{\gamma_{t} E_{t} P V(S)-E_{t} P V(N X)}{E_{t} P V(S)-\left(1-\alpha_{t}\right) E_{t} P V(N X)}\right] \tag{13}
\end{equation*}
$$

1. The nominal exchange rate also reflects two composition effects, $\alpha_{t} \& \gamma_{t}$
2. If $0<\alpha_{t}<1$, exchange rate directly affected by $E P V(S)$ and $E P V(N X)$
3. If $0<\gamma_{t}<1 \& E P V(S)>0, e_{t}$ depreciates as $\gamma_{t}$ rises
4. If $E P V(S)>0$, higher $E P V(N X)$ appreciates home currency
5. If $\gamma_{t}=0$, higher $\alpha_{t}$ appreciates home currency

- so long as $E P V(N X)>0$


## Case Study: Norway

- North Sea oil \& natural gas generating massive revenues
- petroleum sector 50\% of exports \& over 20\% of GDP
- petroleum tax payments by companies 30\% of govt revenues
- Norway's fiscal rule
- all govt revenues from oil transferred to Government Pension Fund Global (GPFG)
- government spending limited to expected real return-4\%-from fund
- all the fund's capital invested abroad


## Norway \& This Framework

1. $B^{G}>0$ gross debt $\approx 20 \%$ GDP \& shrinking
2. $J$ large: GPFG $\approx 130 \%$ GDP \& growing
3. $B_{t}^{G}-e_{t} J_{t}<0$ \& growing
4. Share of foreign assets held by HHs, $\alpha_{t}<1$, maybe negative \& shrinking(?)
5. Share of govt debt held by HHs, $1-\gamma_{t}>0$
6. $\frac{B_{B}^{G}-e_{t} J_{t}}{P_{t}}=E P V(S)<0$ : expect primary budget deficits!
7. Need $E P V(N X)$ to be negative

$$
\begin{aligned}
P_{t} & =\frac{\left[\left(1-\gamma_{t}\right)-\gamma_{t} \alpha_{t}\right] B_{t}^{G}}{E P V(S)-\left(1-\alpha_{t} E P V(N X)\right.}>0 \\
& \Rightarrow\left(1-\alpha_{t}\right) E P V(N X)<E P V(S)<0
\end{aligned}
$$

8. $J \uparrow \& B^{G} \downarrow \Rightarrow$ appreciates currency
9. $E P V(S)<0 \& E P V(N X)<0$ changes comparative statics

- higher $E P V(S) \Rightarrow$ appreciates currency
- higher $E P V(N X) \Rightarrow$ depreciates currency


## Norway \& This Framework

- Growing govt holdings of foreign assets exerts deflationary pressures
- If fiscal policy is passive, Norges Bank can retain control of price level
- are oil revenues directly controlled by government?
- are non-oil surpluses adjusted to stabilize debt?
- Perhaps need to rethink meaning of "passive fiscal policy" when $B_{t}^{G}-e_{t} J_{t}<0$
- Norway presents some interesting challenges to conventional thinking


## Case Study: Japan

- Gross government debt in Japan over 200\% GDP
- much of JGB held by government \& "quasi-government" entities $\Rightarrow B^{G}$ may be much smaller
- little of JGB held by Japanese households \& foreigners
- $\gamma_{t}$ may be quite large
- BoJ owns over \$1 trillion in foreign reserves
- J large, so $\alpha_{t}$ small
- Every study concludes Japanese FP "unsustainable"
- yet interest rates extremely low \& JGBs riskless
- Until recently...
- $¥$ strong \& steady
- chronic trade surpluses
- inflation extremely low (deflation)
- Many puzzles


## Wrap Up

- A great many good research questions remain about policy interactions
- Theory:
- central bank balance sheet operations require fiscal backing
- price-level determination in a monetary union
- open-economy dimensions
- Empirics:
- can the observational equivalence be broken?
- are there historical episodes that look like Regime F?
- need new data sets: market value of debt, maturity structure, real discount rates, primary surpluses

