KALMAN FILTER EXERCISE

(1) Using the same log of quarterly US GDP 1947-2017 data (sorry about not making it clear to use logs in the previous exercise) that you used in the previous exercise, fit an MA(2) model to the first differences of log GDP. That is, estimate by maximum likelihood the model

$$\Delta y_t = a + b\varepsilon_t + c\varepsilon_{t-1} + d\varepsilon_{t-2}\,, \tag{1}$$

where $c\varepsilon_t$ is the innovation in $\log y_t$ and ε_t is i.i.d. $N(0,1)$. Use the Kalman filter to evaluate the likelihood, imposing that the distribution of the initial state is the unconditional distribution implied by the model. Note that, because you are evaluating the full sample likelihood, convergence to a non-fundamental representation is possible, and even likely if you start the maximization far from the parameters of the MLE.

(2) Draw an MCMC sample from the posterior on the parameters and use it to form a histogram for the 10-year annual growth rate of GDP starting at the end of the sample. Compare this to what you found in the previous exercise with an AR(2) model.

(3) In this sample and model, the smoothed and filtered time paths for the state are almost identical. Use your Kalman filter estimates of the covariance matrix of the state to explain why.

R code to evaluate the likelihood with the Kalman filter is available in the exercise directory. You can use it as a guide if you work in another language. If you work in R, the package VARex5132017, available on the exercise directory as a zip file, may be useful. It contains an optimization program, `csminwelNew`, that you can use for the MLE part of the exercise and allows access to easier-to-read documentation for `kf2`, the Kalman filter program. To install the package, download and decompress the zip file, then use the **`install.packages`** command with `repos=NULL`.

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