REVIEW QUESTIONS
NOW WITH SOME ANSWER SKETCHES

(1) A discrete-time univariate process x has the spectral density $S_x(\omega) = e^{-1/\sin^2(\omega)}$. What is its variance? (You probably have to do this as a numerical integral.) Prove that it is possible to forecast this process two steps ahead, based on a finite number of past values of x, with error variance less than one per cent of the unconditional variance. Explain how you would find the coefficients to use in such a predictor. (or actually do it — not too hard to do for a takehome).

The variance of a stationary process is the integral of its spectral density from 0 to 2π, divided by 2π. So you'd need to integrate the given spectral density function numerically.

A linearly regular discrter time process with spectral density S_x satisfies

$$\int_0^{2\pi} \log(S_x(\omega))d\omega > -\infty\,.$$

But $e^{-1/\sin^2(\omega)}$ does not satisfy this condition. It goes to 0 so fast near 0 and 2π that its log has integral $-\infty$. So the process is not linearly regular. That means it can be forecast, with any desired of root mean squared error, arbitrarily far ahead. To construct such predictors, one could inverse-transform back to the time domain to get the autocovariance function. Then choose a large number of lags and use the autocovariance matrix to populate $X'X$ and $X'Y$ matrices for a regression. If the result doesn't produce small enough RMSE, redo with more lags until success is achieved.

I actually did the calculations and the resulting autoregressive coefficient sequences are interesting — showing large and persistent low frequency oscillations.

(2) Primiceri suggests a way to use dummy observations to implement a prior belief that certain cointegrating vectors (e.g. those imposing stability on real, vs. nominal, linear combinations) are likely. How could you extend Primiceri's idea to a model with both nominal interest rates and price levels, where inflation, not just the price level, is likely to have a unit root and interest rates are likely to be cointegrated with inflation?

(3) Show that in a linear regression with the usual Gauss-Markov assumptions but errors with a t distribution with finite degrees of freedom, The posterior distribution on the parameters converges to normality, but with a different covariance matrix from that implied by the standard normal linear model. Would use of the sandwich covariance matrix just bring us back to the usual covariance matrix of OLS in this case?

Depends on whether the t distribution for the errors is a correct assumption or not. If it is, the sandwich will, asymptotically, give the correct covariance matrix, which is also what one gets from the second derivative of the log likelihood using the assumed t distribution. This will be smaller than the OLS covariance matrix, because it is correctly using a non-normality assumption on the residuals. But if, say, the residuals are actually normal,

Date: May 18, 2015.

The sandwich estimator will give a larger covariance matrix, asymptotically, than that of OLS. The t distribution assumption amounts to using weighted least squares, with larger residuals down-weighted, and this is inefficient if the residuals are actually normal.

(4) In the old Sargent-Sims dynamic factor model paper there were two types of factor model: "unobservable index" and "observable index". The "unobservable index" model corresponds to what have recently been called dynamic factor models. The observable index model postulates an autoregressive representation like this:

$$\underset{n\times 1}{y}(t) = \underset{n\times k}{A}(L)\underset{k\times n}{B}(L)y_t + \varepsilon_t\,,$$

with $B_s = 0$ for $s \leq 0$, ε with diagonal autocovariance function, and $k \ll n$. Is this model equivalent to the usual dynamic factor model? A special case of it? They are certainly not equivalent if you require the same k in both models, but the question is interesting if you allow different k's.

Any spectral density that can be written as a diagonal matrix plus a less than full rank positive semi-definite matrix of constant rank is the spectral density of a factor model process. Observable index models can produce such a spectral density matrix. If $k < n/2$, they generally imply the spectral density matrix is a diagonal plus a less than full rank matrix, but the less than full rank matrix might not be positive definite. The possibly most trivial example:

$$x_t = \varepsilon_t$$
$$y_t = x_{t-1} + \nu_t$$

which is an observable index model equivalent to the unobservale index model

$$x_t = f_t$$
$$y_t = f_{t-1} + \nu_t$$

(5) In our exercise on breaks, the log likelihood rose as the break date approached the beginning or end of the sample. (This was not visible for the likelihood itself, because the likelihood was so much higher near the likelihood peak.) This tendency to for likelihood to rise approaching the beginning or end of the sample is actually to be expected. Why?