EXERCISE ON WOLD DECOMPOSITION, ARMA MODELS

(1) In all of the models below, ε_t is i.i.d. $N(0,1)$ and y_t is a stationary process. Though they have different coefficients, not all of them define different distributions for y_t. Which pairs define identical stochastic processes?

$$y_t = .7y_{t-1} + \varepsilon_t - .9\varepsilon_{t-1} \tag{1.1}$$

$$y_t = 2.9y_{t-1} - 2.8y_{t-2} + .9y_{t-3} + \varepsilon_t - 2\varepsilon_{t-1} + \varepsilon_{t-2} \tag{1.2}$$

$$y_t = .9y_{t-1} + \varepsilon_t - .7\varepsilon_{t-1} \tag{1.3}$$

$$y_t = 1.7y_{t-1} - .95y_{t-2} + .175y_{t-3} + \varepsilon_t - 1.9\varepsilon_{t-1} + 1.15\varepsilon_{t-2} - .225\varepsilon_{t-3} \tag{1.4}$$

$$y_t = .9y_{t-1} + \varepsilon_t \tag{1.5}$$

(2) The stochastic process y is a finite-order moving average of the form

$$y_t = \varepsilon_t + 1.7\varepsilon_{t-1} + .7\varepsilon_{t-1}$$

with ε_t i.i.d. $N(0,1)$.

(a) Show that this process does not have an AR representation, even if we allow infinitely many lags.

(b) Determine the form of the best predictors of y_t based on 2, 5, and 10 lagged values of y and compare the forecast error variance of these approximations to the forecast error variance of $E[y_t \mid \{y_s, s = -\infty, \dots, t-1\}]$. [You'll need to use a computer. The `toeplitz` functions in R and Matlab should be helpful.]

(3) Again all the models below have i.i.d. $N(0,I)$ ε processes, and in each, y is stationary. For each model, find the fundamental MAR and compare the variance of the innovation to the variance of ε.

$$y_t = \varepsilon_t + 2\varepsilon_{t-1} + 1.1\varepsilon_{t-2} \tag{3.1}$$

$$y_t = \varepsilon_t - 2\varepsilon_{t-1} + .99\varepsilon_{t-2} \tag{3.2}$$

$$y_t = \varepsilon_t + \begin{bmatrix} 0 & 5 \\ 0 & 0 \end{bmatrix} \varepsilon_{t-1} \,. \tag{3.3}$$

In the first two of these, y is scalar, while in the last, both y and ε are two-dimensional.

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