## 1. EXERCISE DUE MONDAY, 9/28

Use monthly data on total nonfarm payroll employment from 1939 to the present. We will use  $y_t$  to refer to the natural log of employment at date t. We want to separate a seasonal from a non-seasonal component of this series. Our model is

$$y_t = s_t + b_t$$
  
 $s_t = s_{t-12} + \varepsilon_t$   
 $b_t = 1.9b_{t-1} - .9b_{t-2} + \nu_t$ .

Define a state vector that lets you cast this model in the form of the standard Kalman filter state evolution and observation equations. Take the standard deviation of  $v_t$  to be .004 and that of  $\varepsilon_t$  to be .001. As is standard for the Kalman Filter, treat  $\varepsilon_t$ ,  $v_t$  as i.i.d. across time and uncorrelated with lagged states. Here we assume they are also mutually uncorrelated at each t. You will need 12 values of  $s_t$  in the state vector. Take their initial covariance matrix to be the matrix with typical element in the t'th row, j'th column  $200 \cdot .96^{|i-j|}$ . Take the initial covariance matrix of the two value of b you need in the state vector to be the matrix with 200 on the diagonal and 196 in the off-diagonal position. Take the initial values of s and b to be uncorrelated in the prior.

Apply the Kalman filter. Plot the filtered estimates of  $b_t$  over the whole sample. For dates after 1947, plot filtered  $b_t$  and  $y_t$  on the same graph. (You might want to adjust the level of  $\hat{b}_t$  so it matches  $y_t$  in January 1948, e.g.) Plot filtered  $s_t$  the last two years in the sample and for 1950 - 51. Does it look like the seasonal pattern has changed much? Plot  $\hat{s}_t$  and  $y_t$  on the same graph for the last four years of the sample, adjusting the level of  $\hat{s}_t$  so it matches  $y_t$  in January 2006. How much of the variation in the data is due to the seasonal?

The payroll employment data and code for one iteration of the Kalman filter, in R and in Matlab versions, are on the course website. The data are in both CSV format, which can be read in to excel, Matlab (I think) or R (via read.csv()) and .RData format, which can be read in to R with the load() command. The latter file, when read in, will put an R time series object y in your workspace that is logged payroll employment and has dates associated with it.

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