FISCAL POLICY AND THE PRICE LEVEL

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These notes are missing interpretation of the results, and especially toward the end, skip some steps in the mathematics. But they should be useful in providing background for the lecture Sunday.

In many standard models of monetary economies, the equilibrium is fragile. That is, there is an equilibrium in which the price level is stable over time when monetary policy takes what is usually thought to be the appropriate form, but there are also many other equilibria. If the existence of the other equilibria is noted at all, they are usually casually dismissed as somehow less interesting. (An example is the textbook *Models of Monetary Economies* (Champ, Freeman and Haslag, 2011.) Here is a model that displays this point. It is exactly the Samuelson pure consumption loan model, except that we will have interest paid on the "money" and call it "debt".

There is an infinite sequence of periods, in each of which the same number of two-period-lived agents is born and endowed with one unit of the consumption good, grain. The grain can be stored, but decays in storage by a factor θ. There is also government debt, denominated in dollars. Its amount at the initial date $t = 1$ is B_0, and it is held by the initial old, who redeem it with the government, receiving in return new one-period debt in the amount $B_1 = R_0 B_0$. Since this new government paper is worthless to the initial old, they attempt to sell it to the initial young, for grain. The price level at date t is the rate at which grain trades for newly issued government debt. This process repeats thereafter for $t = 1, \ldots, \infty$.

Formally, the generation born at t maximizes its lifetime utility $U(C_{1t}, C_{2,t+1})$ subject to the constraints

$$C_{1t} + S_t + \frac{B_t}{P_t} = 1 \tag{1}$$

$$C_{2,t+1} = \frac{R_t B_t}{P_{t+1}} + \theta S_t \tag{2}$$

$$S_t \geq 0\,, \qquad B_t \geq 0\,. \tag{3}$$

Because the government is doing nothing but rolling over the debt each period, the market clearing condition is simply $R_t B_t = B_{t+1}$. The government sets an arbitrary

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value for R_t each period. The first-order conditions for an agent in generation t, assuming perfect foresight about next period's P, are

$$\partial C_1 : \qquad D_1 U(C_{1t}, C_{2,t+1}) = \lambda_t \tag{4}$$

$$\partial C_2 : \qquad D_2 U(C_{1t}, C_{2,t+1}) = \mu_{t+1} \tag{5}$$

$$\partial B_t : \qquad \frac{\lambda_t}{P_t} = \frac{R_t \mu_{t+1}}{P_{t+1}} \text{ , if } B_t > 0 \tag{6}$$

$$\partial S_t : \qquad \lambda_t = \theta \mu_{t+1} \text{ , if } S_t > 0\,. \tag{7}$$

The B and S first order conditions tell us, as we would expect, that if agents are storing grain and also buying debt, their returns must match, so that in that case

$$\frac{R_t P_t}{P_{t+1}} = \theta \tag{8}$$

In order that we can get easily computed solutions that give us some insight into how the model works, we assume R_t is constant and

$$U(C_{1t}, C_{2,t+1}) = \log(C_{1t}) + \log(C_{2,t+1})\,.$$

Then the Lagrange multipliers can be solved out to deliver

$$\frac{R_t P_t}{P_{t+1}} = \frac{C_{2,t+1}}{C_t} \text{ , if } B_t > 0 \tag{9}$$

$$\theta_t = \frac{C_{2,t+1}}{C_t} \text{ , if } S_t > 0\,. \tag{10}$$

Let savings be represented by $W_t = S_t + B_t/P_t$. Logarithmic utility makes solution easy because it implies that whatever the rate of return to savings, call it ρ_t, we will have $\rho = C_{2,t+1}/C_{1t}$, and this in turn implies that

$$C_{1t} + W_t = 1 = C_{1t} + \frac{C_{2,t+1}}{\rho_t} = 2C_{1t}\,. \tag{11}$$

Thus savings is always half the endowment, i.e. .5.

This economy has an equilibrium in which there is no storage and nominal debt has value (i.e. $P_t < \infty$). With no storage, $C_{1t} = .5$ and, since savings is all used to buy debt from the older generation, $C_{2t} = .5$ also. This means $\rho_t \equiv 1$ and therefore $R = P_{t+1}/P_t$, all t. In other words, the price level grows at the gross interest rate and the real value of both newly issued and maturing debt is constant at .5. In order for this equilibrium to prevail, the initial price level P_1 must be $2B_1$, i.e. $2RB_0$.

The economy also has equilibria in which $S_t > 0$, however. In these equilibria, of course, $\rho \equiv \theta = RP_t/P_{t+1}$. In other words, The price level grows not at the rate R, but at the higher rate R/θ. The nominal debt still grows at the rate R, however, so the real debt shrinks over time, with

$$\frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}}\,.$$

The economy can start with any $B_1/P_1 < .5$. Storage will then be $S_1 = .5 - B_1/P_1$. In subsequent periods, S_t increases toward .5 as the real value of savings in the form of nominal bonds shrinks toward zero.

In other words, *every* initial price level P_1 that exceeds $2RB_0$, including $P_1 = \infty$ (in which case bonds are valueless and all savings is in the form of storage), corresponds to a perfect-foresight equilibrium in this economy. This is an economy with an indeterminate price level.

Note that the economy's resource constraint is $C_{1t} + C_{2t} + S_t = 1 + \theta S_{t-1}$: consumption and storage by the young plus consumption by the old is endowment of the young plus the proceeds from storage by the old. Since in all the equilibria with positive storage S is either increasing or (when $P_t = \infty$) constant, and since $C_{1t} \equiv .5$, $C_{2t} < .5$ in all these equilibria with $S_t > 0$. That is these equilibria with $S_t > 0$ are strictly worse than the one in which $S_t = 0$. It may be comforting to believe that somehow these worse equilibria would be avoided, but there is nothing in the structure of the model that should make the worse equilibria less likely.

When we use the "B" and "R" notation as here, it is perhaps unsurprising that we get an indeterminate equilibrium when the government issues debt without any backing from taxation. But if we replace B by M and set $R = 1$, this becomes Samuelson's model of "money" and is sometimes taken as a useful metaphor to aid understanding of how fiat money can have value.

But back to thinking of it as debt. What if we do provide tax backing for the debt? Suppose everything is as before, but now the government imposes a lump sum tax τ on the old each period. The government budget constraint is now

$$\frac{B_t}{P_t} = \frac{RB_{t-1}}{P_t} - \tau \,. \tag{12}$$

Suppose there were an equilibrium in which savings is in the form of both bonds and storage. Then both must have real gross rate of return θ. That makes the government budget constraint

$$\frac{B_t}{P_t} = \theta \frac{B_{t-1}}{P_{t-1}} - \tau \tag{13}$$

This is a stable difference equation in B_t/P_t. If it starts operation at $t = 0$, we will have

$$\frac{B_t}{P_t} = \sum_{s=0}^{t-1} -\tau\theta^s + \theta^t \frac{B_0}{P} \,. \tag{14}$$

But notice that, since $\theta < 1$, the right-hand side of this expression eventually becomes negative, converging as $t \to \infty$ to $-\tau/(1-\theta)$. That is, if the economy started on a path satisfying this condition, eventually it would reach a point where the government is putting grain in the amount τ on the market to exchange for mature debt, but no one would have any debt to exchange for it. Anyone foreseeing this would have a motive for holding on to some debt to exchange for grain at an extremely favorable price ratio when everyone else had run out. So these paths cannot be equilibria. By imposing the tax, no matter how small, the government has eliminated all those equilibria in which storage and debt coexist. It has also eliminated the equilibria in which debt is valueless ($P_t = \infty$) for the same reason: the government is trying to exchange τ units of grain per capita for mature debt, so th mature debt is necessarily of some value.

If there is no storage, the rate of return on debt can be positive. The tax is recognized by individual agents as reducing their wealth, so first-period consumption is reduced. Formally, the private budget constraint in the second period is now

$$C_{2,t+1} = \frac{RB_t}{P_{t+1}} - \tau \,. \tag{15}$$

We will still have, from the first-order conditions, $RP_t/P_{t+1} = \rho_t = C_{2,t+1}/C_t$, where ρ_t is just notation for the real rate of return. Using these last two expressions to rewrite the first-period budget constraint, we have

$$C_{1t} + \frac{C_{2,t+1} + \tau}{\rho_t} = 1 = C_{1t} + \frac{\rho_t C_{1t} + \tau}{\rho_t} \,. \tag{16}$$

With no storage, $C_{1t} + C_{2t} = 1$, which implies

$$C_{1t} + \rho_{t-1} C_{1,t-1} = 1. \tag{17}$$

Solving this equation for ρ_{t-1} and substituting back into (16) gives us

$$2C_{1t} + \tau \frac{C_{1t}}{1 - C_{1,t+1}} = 1 \,. \tag{18}$$

This reduces to the equation

$$1 - C_{1,t+1} = \frac{\tau C_{1t}}{1 - 2C_{1t}} \,. \tag{19}$$

Solving this equation for a steady state with $C_{1,t+1} = C_t$ produces a quadratic equation whose solution is

$$C_{1t} \equiv \frac{3 + \tau - \sqrt{1 + 6\tau + \tau^2}}{4} \,. \tag{20}$$

There is a second root to the quadratic equation, but it puts C_{1t} above one, which is impossible. For small values of τ, the solution can be approximated, via a Taylor expansion, as

$$C_{1t} \equiv \frac{1 - \tau}{2} \,. \tag{21}$$

There is always a positive steady state between zero and one when $\tau > 0$, and $C_{1t} \to 0$ as $\tau \to \infty$.

If C_{1t} begins at a value other than its steady state value, equation (20) implies that C_{1t} eventually takes on impossible values. Agents foreseeing that will rule out any deviations from the steady state. In other words, with $\tau > 0$, The model has a unique, constant solution for C_{1t}. Since $B_t/P_t = 1 - C_{1t}$, there is also a unique, constant solution for real debt $B_t/P_t = RB_{t-1}/P_t$. With this equation holding at $t = 1$, the initial price level P_1 is uniquely determined. Since for every value of τ, $C_{1t} < \frac{1}{2}$, $C_{2t} = 1 - C_{1t} > \frac{1}{2}$, and therefore the real rate of return on government debt exceeds one.

The optimal allocation is that achieved in the non-unique steady state equilibrium without taxes, where $C_1 = C_2$ at all dates. So these equilibria with $\tau > 0$ deliver lower utility than that equilibrium. But for small values of τ, the distortion is slight, as can be seen from (21). Notice that the real value of debt does not go to zero as $\tau \to 0$. Instead it converges to $\frac{1}{2}$.

The government budget constraint, solved forward, implies that with a constant real interest rate $\rho > 1$ (as in this model's stationary equilibria for $\tau > 0$), we can solve forward to obtain

$$\frac{B_t}{P_t} = \frac{\tau}{\rho - 1}, \tag{22}$$

i.e. the real value of the debt is the discounted present value of future lump sum taxes τ. As $\tau \to 0$, $\rho \to 1$, so that the ratio of τ to $\rho - 1$ converges to $\frac{1}{2}$.

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