

## NOTES ON SEIGNORAGE

Seignorage is raising government revenue by expanding the money supply. It helps to think of it in terms of three budget constraints: The central bank's (which we'll call the "Fed"), the Treasury's, and the consolidated government, where the Treasury and the Fed are combined into a single "government" constraint.

### Variables:

- $B$  total government debt issued by the Treasury
- $B^F$  government debt held by the Fed
- $B^P$  government debt in the hands of the public
- $S$  payments by the Fed to the Treasury (one definition of "seignorage")
- $\tau$  real primary surplus
- $R_t$  gross nominal interest rate on debt issued at  $t$
- $M$  high-powered money (currency plus reserve deposits at the Fed)

$$\text{Treasury :} \quad B_t = R_{t-1}B_{t-1} - \tau_t P_t - S_t \quad (1)$$

$$\text{Fed :} \quad B_t^F + S_t = R_{t-1}B_{t-1}^F + M_t - M_{t-1} \quad (2)$$

$$\text{Government :} \quad B_t^P = R_{t-1}B_{t-1}^P - \tau_t P_t - M_t + M_{t-1} \quad (3)$$

Note that the consolidated government constraint (3) is just the Treasury constraint minus the Fed constraint. The Fed constraint is subtracted because for it  $B^F$  is an asset, whereas for the Treasury  $B$  is a liability. The  $S_t$  terms drop out, as they just reflect a transfer within the government. In the consolidated government constraint the term  $M_t - M_{t-1}$  enters just as does the nominal primary surplus  $\tau_t P_t$ , so we can think of  $\Delta M_t = M_t - M_{t-1}$  as a source of revenue. This term  $\Delta M_t$  is another definition of seignorage.

We can rewrite the consolidated government constraint defining the government's total liabilities as  $B_t^P + M_t = A_t$ . Then it becomes

$$A_t = R_{t-1}A_{t-1} - \tau_t P_t - (R_{t-1} - 1)M_{t-1}. \quad (4)$$

The last term on the right of this equation can be thought of as the interest the government avoids having to pay by having some of its debt in the form of non-interest-bearing high-powered money. This is yet another definition of seignorage. So we have three definitions:  $S_t$ ,  $\Delta M_t$ , and  $(R_{t-1} - 1)M_{t-1}$ .

## 1. THE OLDEST FORM OF SEIGNORAGE

The government makes coins from gold. It may start by just issuing standardized weights of gold coins, so that a coin of a given size and shape represents a given weight of gold. But if the coins circulate at par, it will be tempting to debase the alloy or shave a little of the weight off the coins. The king then makes a profit by buying gold with coins that contain slightly less gold than the gold being bought.

## 2. THE INFLATION TAX

Seignorage is sometimes referred to as an “inflation tax”. In the definitions above —  $S_t$ ,  $M_t - M_{t-1}$ , and  $(R_{t-1} - 1)M_{t-1}$ , inflation has not appeared. We can understand the idea of an inflation tax if we rewrite the total government budget constraint in terms of real debt  $b_t = B_t/P_t$  and real high-powered money  $m_t = M_t/P_t$ :

$$b_t = \frac{R_{t-1}P_{t-1}}{P_t}b_{t-1} - \tau_t - m_t + m_{t-1}\frac{P_{t-1}}{P_t}. \quad (5)$$

Here we see that, with real debt and real money balances held constant, inflation eases the government budget constraint in two ways: It shrinks the real value of the interest-bearing debt brought over from the previous period, and it shrinks the real value of the high-powered money carried over as a liability from the last period. Putting this another way, inflation creates room for the government to issue new nominal debt and new money, thereby acquiring real resources, without increasing the real value of debt or money.

But this analysis of the effect of inflation on the government budget neglects the fact that if the inflation is *anticipated*, buyers of debt will insist on  $R_t P_t / P_{t+1} = \rho_t$ , where  $\rho_t$  is the real interest rate (real return on other assets) prevailing at time  $t$ . So if we hold the real rate constant and assume the inflation is anticipated, the constraint reads as

$$b_t = \rho_t b_{t-1} - \tau_t - m_t + m_{t-1} \frac{P_{t-1}}{P_t}. \quad (6)$$

In other words, if inflation is anticipated, all that is left of the inflation tax is the effect of inflation on  $m_t - m_{t-1}P_{t-1}/P_t = \Delta M_t/P_t$ . The difference between the inflation tax with unanticipated inflation and the tax with anticipated inflation is very large for an economy like that in most rich countries, where interest-bearing debt is much larger than the real value of high-powered money.

## 3. CENTRAL BANK PROFITS

At any given date,  $S_t$ , the amount the central bank transfers to the Treasury, has no necessary relation to either of the other two measures of seignorage. But if the

bank has zero capital, so its assets  $B_t^F$  exactly match its liabilities  $M_t$  at all dates, then we can see from its budget constraint (2) that  $S_t = (R_{t-1} - 1)M_{t-1}$ , i.e. the transfer to the Fed is the “interest savings” measure of seignorage.

#### 4. MAXIMIZING SEIGNORAGE REVENUE

In a steady state with constant, anticipated inflation, we have seen that seignorage is  $m_t - m_{t-1}P_{t-1}/P_t$ . If also  $m_t$  is constant at  $m$ , seignorage is  $m(1 - P_t/P_{t+1})$ . But anticipated inflation increases nominal interest rates and thereby reduces demand for real balances, so increasing inflation does not always increase steady state seignorage revenue. You worked out in an exercise the revenue-maximizing rate of inflation when  $m_t = R^{-\frac{1}{2}}$ , for example.

#### 5. INTEREST ON RESERVES

Most central banks, including the Fed since 2008, now pay interest on reserve deposits. Usually they pay a little less than the interest rate on government debt, but now in the US the rate on short term treasury bills and on reserve deposits are nearly the same (with reserve deposits paying *more*). If reserve deposits pay interest, they are a form of short-term government debt, and from the point of view of the government as a whole, an open market operation that buys short-term Treasury bills, paying for them with reserve deposits, is having no effect on the overall government fiscal balance. One kind of short term debt is simply replacing another. The Treasury can still be thought of as selling debt to the Fed, but now instead of this replacing an interest-bearing liability with a non-interest-bearing one, it simply converts the Treasury debt into a Fed liability, paying the same rate. One can still calculate  $\Delta M_t$  and call it “seignorage”, but it is no longer a form of taxation.

The extreme case is when both reserves and currency pay interest, and at the same rate as the rate on interest-bearing Treasury debt. Then the two forms of government liability,  $B$  and  $M$ , are equivalent, and the version of the government budget constraint that consolidates them (4) would become simply  $A_t = R_{t-1}A_{t-1} - \tau_t$ . Surprise inflation could still produce an inflation tax by reducing the real value of  $A_{t-1}R_{t-1}$ , but anticipated inflation no longer produces any “tax” to relax the government budget constraint,

Of course even with interest on reserves, a substantial part of the Fed’s liabilities are currency, which does not pay interest. Also, in normal times, central banks keep the interest rate on reserves somewhat below that on Treasury debt. So inflation does still act as a tax, but the implicit revenue from anticipated inflation is much reduced by the payment of interest on reserves at near-market rates.

## 6. SUMMARY

- Seignorage is government purchasing power obtained through expansion of the money supply.
- It can be measured as  $\Delta M_t$ , money growth, or as  $(R_{t-1} - 1)M_{t-1}$ , interest savings to the government of having non-interest-bearing liabilities.
- These two measures are nearly the same when inflation is anticipated.
- The “inflation tax” is nearly the same as either of these definitions of seignorage when inflation is anticipated.
- When inflation is unanticipated, the “tax” is levied on interest-bearing as well as non-interest-bearing liabilities and is thus much larger than seignorage itself.
- To the extent that interest is paid on high-powered money the inflation tax is reduced, and may be much smaller than the conventional  $\Delta M_t / P_t$  measure of seignorage.