Multiple equilibria

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Why did the interest rate on Italian debt jump?

- The ratio of Italian government debt to GDP is high, but it has been high for a long time.
- The primary surplus in Italy has been adequate to pay the interest costs on its debt, so its debt was not growing rapidly.
- But recently the interest rate on Italian government debt has jumped up.
- One explanation: self-fulfilling expectations of default.

Self-fulfilling expectations?

- The idea: If bondholders do not expect default, the interest rate stays low, primary surpluses are adequate to service the debt, and there is no default.

- But if bondholders do expect default, they will demand a higher interest rate, primary surpluses are inadequate, and default becomes likely.

- So if a central bank could guarantee that no default would occur, the interest rate would stay low, and the guarantee would never be invoked.

- We'll first stick to the case of "real" debt, i.e. debt denominated in gold, euros, or (for countries other than the US) dollars.

Case I: No "fundamental" problem

- Primary surplus is τ, now and forever.
- Required expected real return is ρ.
- Primary surplus will support current debt forever with no default risk (i.e. with $R_t \equiv \rho$):

$$b_t = \bar{b} = R_{t-1}b_{t-1} - \tau = \rho b_{t-1} - \tau = \rho\bar{b} - \tau$$

- This means $\bar{b} = \tau/(\rho - 1)$.

A high interest rate $R > \rho$, with τ constant, would make debt steadily rise,

- Now suppose that bondbuyers become nervous, think there might be a default, so require an interest rate $R > \rho$ in order to hold debt.
- If $b_t = \bar{b}$, then $b_{t+1} = R\bar{b} - \tau > \rho\bar{b} - \tau = \bar{b}$.
- So long as $b_t > \bar{b}$ and $R > \rho$, $b_{t+1}/b_t > 1$, and b_{t+1}/b_t increases as b_t increases. **Exercise A: Prove this.**
- Debt goes above $\bar{b}$ and keeps growing without bound.

There is an upper bound on b_t

- Bond buyers will not hold arbitrarily large amounts of real debt b_t.
- So the $R > \rho$, $\tau_t \equiv \tau$, $b_t > \bar{b}$ situation cannot go on forever.
- Either

  a. b and R drop back to sustainable levels, or
  b. τ adjusts upward, to an amount that can service the higher debt.

Default

- A downward movement in b_t, with τ constant, constitutes default.

- If there is a firm commitment to $\tau_t \equiv \tau$ forever (possibly because higher revenues are impossible), the high interest rates generated by fear of default make default inevitable.

- But knowing this, would a rational investor every buy a government bond, even at a high interest rate?

If default date is known

- Suppose default means the value of the debt reverts at t to $b_t = \bar{b}$, and that $b_{t-1} > \bar{b}$.

- This means that R_{t-1} is irrelevant to the return from $t-1$ to t. The return will actually be $\bar{b}/b_{t-1}$. This is below one.

- So no one will buy the debt, at any quoted interest rate, at $t-1$. But this means the government will not be able to roll over its debt at $t-1$, and will actually have to default then.

- And then the same holds for $t-2, t-3$, etc, back to the initial date.

Instant default?

- In other words, default will occur immediately as soon as interest rates rise because of fear of default.
- But in our setup, default means reversion of the value of debt to $\bar{b}$. Since at the initial date $b_t = \bar{b}$, default is not an issue.

Random default

- Suppose again that default means reversion of the value of debt to $\bar{b}$, with there being no worry of additional defaults after that.

- Then default with, say, a 10% probability each period can be ruled out by a similar argument.

Default with consequences

- Now suppose that some of the debt is bought by banks, that the banks finance purchase of the debt by taking deposits, and that the government will bail out insolvent banks if their assets suddenly lose value.

- This implies that at the time of default, the government will make a large expenditure, say some fraction ϕ of $b_t - b_{t-1}$ at the date of default.

- If they are going to be bailed out, the banks find the government bonds non-risky, so they are happy to hold them despite default risk. In fact we have to assume they face some constraint (maybe reserve or capital requirements) that keeps them from buying all the bonds.

- If the date of default is known, as before no one would hold the debt in the previous period, which forces the date of default back to the initial date.

Non-trivial default

- But now, the discounted present value of future surpluses in the event of default will be less than $\bar{b} = \tau/(1-\rho)$, because of the bailout.

- So default is not trivial, and it occurs instantly if at all.

- Same conclusion for randomlly timed default in this case.

General conclusion

- Pure multiple equilibrium, based on expectations only, is unlikely if agents are forward looking.

- But if default triggers a decline in future primary surpluses, multiple equilibria are possible.

- Default occurs right away, as soon as worries about it arise, if agents are forward-looking.

Flexibility from nominal debt

- If there is a price level that varies, and debt is nominal, "default with consequences" need not occur immediately, once anticipated.
- Suppose primary surplus before default is τ_0, after default is $\tau_1 < \tau_0$.
- Every period probability of non-default is π, expected real return on debt is ρ.
- Then $E_t b_{t+1} = \rho b_t - (\pi \tau_0 + (1 - \pi)\tau_1)$.
- Since the future path of τ's looks the same every period, b must be constant. (This is true whether debt is real or nominal.)

The difference

- Actual, as opposed to expected, debt in the real-debt case when default does not occur, satisfies $b_{t+1} = R_t b_t - \tau_0$.

- As we observed above, if initial debt is at the level that would be consistent with permanent $\tau_t \equiv \bar{\tau}$, this requires $b_{t+1} > b_t$, which is impossible with forward-looking agents, as we have just observed.

- But with nominal debt, surprise inflation can intervene. If default does not occur, prices fall to keep $b_t = B_t/P_t$ in line with the discounted present value of future expected primary surpluses. If default does occur, in the sense that τ drops to a new permanent level, the drop in the value of b comes through inflation, not formal bankruptcy.

Exercise B

Suppose τ_0 is 10, τ_1 is 8, the probability of switching from τ_o to τ_1 permanently is .1 every period, the amount of nominal debt initially is B_0, and the real gross rate of return is ρ. Find the nominal interest rate that prevails before the drop in τ, the inflation or deflation every period before the drop, and the inflation rate at the date of the drop. Assume that after the drop everyone is certain that τ_1 will prevail forever.