## EXERCISE DUE TUESDAY 11/22 AND DISCUSSION QUESTIONS FOR PRECEPT

(1) There is an investment opportunity that will pay a rate of return 6% with probability .5 and a rate of return -3% with probability .5. You have $1000 of your own money to invest.
   (a) If you simply invest your own money, what is your expected rate of return? the variance of your rate of return?
   (b) Suppose you can borrow an additional $1000, achieving a leverage ratio of 1 to 1, paying an interest rate of one per cent. What would be the distribution of rates of return on the leveraged investment? What would the mean and variance of rates of return be on the leveraged investment?
   (c) For each of the following combinations of borrowing rates of interest and leverage ratios calculate the distribution of rates of return and its mean and variance:
       (i) borrowing rate 1%, leverage ratio 20 to 1.
       (ii) borrowing rate 2%, leverage ratio 1 to 1.
       (iii) borrowing rate 2%, leverage ratio 20 to 1.
   (d) Can you state a general rule for determining from the distribution of rates of return on the unleveraged investment and from the borrowing rate, whether increasing leverage increases expected return?

(2) Suppose the economy is in a steady state in which real balances $M_t/P_t$ are constant and the money stock is growing at the constant gross growth rate $G$, so $M_t = GM_{t-1}$. Suppose the central bank has an old-fashioned central bank balance sheet, so that its liabilities are just $M_t$ and its assets $B_t$ are government bonds paying a gross interest rate $R_t$, with $B_t = M_t$ always (because the bank turns over seignorage earnings to the treasury). Assume that there is a constant gross real interest rate $\rho$ that must match the real return on government debt so that
$$R_t \equiv \rho \frac{P_{t+1}}{P_t} \,.$$
The central bank's profits at date $t$ in real terms are then $(R_{t-1} - 1)B_{t-1}/P_t$.
   (a) Show that in this steady state these profits move one-for-one with the other measure of seignorage, $(M_t - M_{t-1})/P_t$.
   (b) Suppose that the demand for real balances in steady state decreases with the nominal interest rate, according to the money demand equation
$$\frac{M_t}{P_t} = \frac{1}{\sqrt{R_t}} \,.$$
Determine what steady state growth rate $G$ for money maximizes seignorage revenue.

**Discussion questions**
   (i) Under a pure gold standard (almost never seen) the central bank holds actual gold to back every issue of currency. Its assets are all gold and its liabilities are all currency. Does it generate seignorage revenue by expanding the amount of currency? Why or why not?

(ii) Discuss the Stella article: What is a "revaluation account" and why do they appear on many central bank balance sheets? What are examples of central banks that have issued large amounts of debt on their own account? Why do they do that and what problems does it create? Stella gives examples of central bank "recapitalizations" that do not actually improve the bank's situation, because the assets provided by the treasury are of little value. What are examples of these assets and why are they of little value? Stella documents a downward trend in central bank profits in recent years. Why does this raise problems and what might explain the trend?