EXERCISE ON NET WORTH AND LENDING RATES, DUE TUESDAY, 10/25

This exercise has you work through a simplified version of the model in Zha (2001, Section 2, "A Simple Case"). You should read that section of that article. You need not try to follow the mathematics of the more complicated dynamic model in the remainder of the paper, but you should read the discussion of its results, describing the capital accumulation and distributional effects of the bankruptcy exemption level.

Consider a firm that has the opportunity to invest in a project with a random rate of return A. The return is either A_u or A_ℓ, with each having probability .5. The project requires an investment of 1.0 if it is undertaken at all. The firm has wealth W initially. If $W > 1$, the firm can fund the project entirely out of its own wealth. If $W < 1$, the firm must borrow from the bank in the amount $1 - W$ to fund the project. The bank charges a gross interest rate R^*, so that the loan contract requires payment of R^*B if the loan is in the amount B. If the investment is undertaken and $B > 0$, the loan contract specifies that if $A < R^*B$, the bank takes the entire return A and the firm is left with nothing. Because the risk of bankruptcy (i.e. of $R^*B < A$) depends on W, the bank's interest rate R^* will depend on W. There is a given market deposit rate R. The firm has the option of making no investment, taking out no loan, depositing its wealth W in the bank, and receiving a return RW. The bank requires that the expected rate of return on any loan it makes, that is, the expected value of $\min(R^*B,A)/B$, must be R. (We think of the bank as in a competitive banking sector. It must make an expected return of at least R to survive, and competition will force it not to demand an expected return higher than R.) We can represent these conditions formally as follows:

$$B = 1 - W \text{ if } B > 0 \tag{1}$$

$$E\left[\frac{\min(R^*B,A)}{B}\right] = R \qquad\qquad \text{Bank return requirement} \tag{2}$$

$$E\left[\frac{\max(A - R^*B,0)}{W}\right] \geq R \qquad\qquad \text{firm return requirement} \tag{3}$$

(a) Suppose $R = 1.01$, $A_u = 1.1$, and $A_\ell = .96$. Plot $R^*(W)$, the bank's interest rate as a function of firm initial wealth W. Note that initial wealth could be negative Are there levels of wealth at which the bank will refuse to lend at any rate that the firm finds acceptable? Will the firm always want to borrow at the bank's rate $R^*(W)$ so long as $W < 1$?

(b) What if $R = 1.05$ rather than $R = 1.01$?

REFERENCES

ZHA, T. (2001): "Bankruptcy Law, Capital Allocation, and Aggregate Effects: A Dynamic Heterogeneous Agent Model with Incomplete Markets," *Annals of Economics and Finance*, 2, 379–400, http://www.aeconf.net/Articles/Nov2001/aef020205.pdf.