

# **Asset Pricing II: Bubbles, peso problems, Ponzi schemes, etc.**

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## **Why might we see a pattern of high yields, or rapid growth in asset values, followed by a crash?**

- This is an interesting question, because we have seen this in recent years. Is it a sign of malfunctioning markets?
- Might be, or might not. We'll discuss examples.

## Stock pricing with growing dividends

- If dividends are growing over time, say by the factor  $g > 1$  each period, so  $\delta_t = g^t \delta$ , the price of the security is

$$Q_t = \delta g^t \sum_{s=1}^{\infty} \phi^s g^s = \delta g^t \frac{\phi g}{1 - \phi g}.$$

(Note that we'd better have  $g\phi < 1$ , or the sum does not converge.)

- This is sometimes called the “Gordon model” for stock valuation.

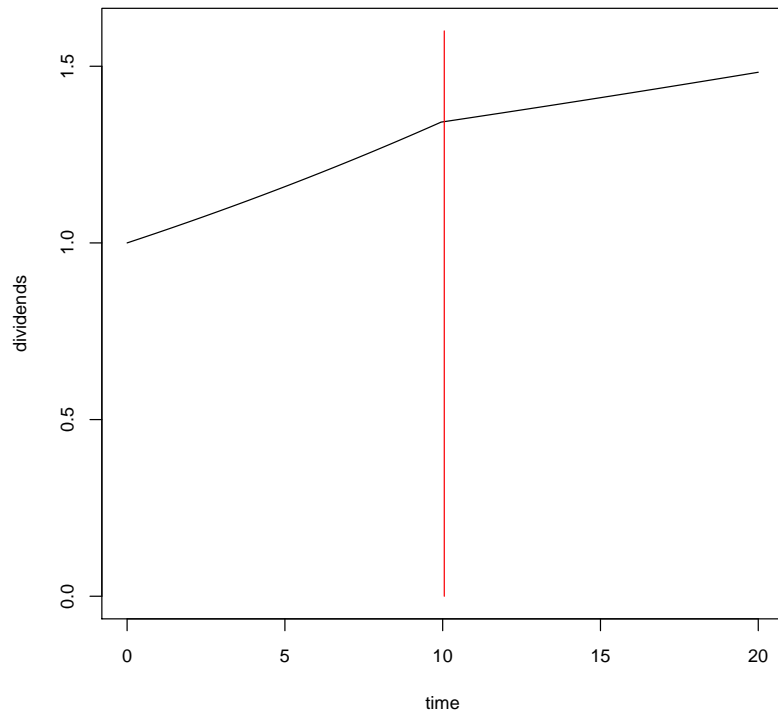
## What if the growth rate drops?

- If at date  $t$  we realized that hence forth growth will only be at the rate  $h < g$ ? The new price is  $\delta g^t \phi h / (1 - \phi h)$ . The ratio of the new to the old price is

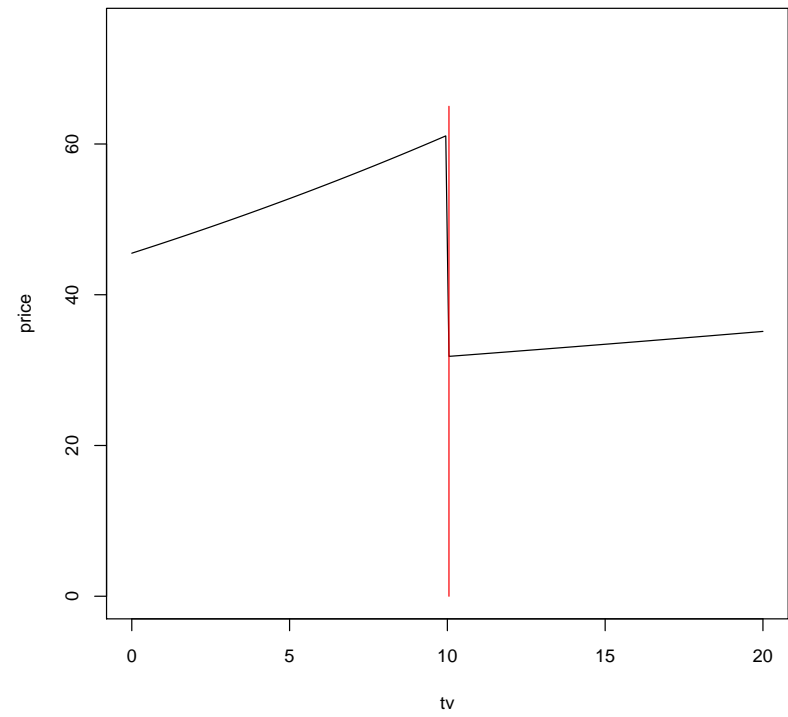
$$\frac{h(1 - \phi g)}{g(1 - \phi h)}$$

- Say  $\phi = .95$ ,  $g = 1.03$ , and  $h = 1.01$ . The ratio is then approximately  $1.01(1 - .98)/(1.03(1 - .96)) \doteq .49$ .
- In other words, a drop in expected growth rate from 3% to 1% causes a price drop of over 50%, as soon as it is recognized, even though dividends themselves remain on a continuous time path.
- This might disturb a lot of investors, but it is not a sign of market malfunction.

**Dividends, with change in growth rate**



**Price with break in dividend growth rate**



## **New idea that may be disproved at any date, or pay off handsomely at $T$**

- Suppose an investment may pay a known amount  $X$  at some future date  $T$ , if the idea behind the investment is correct.
- But each period before  $T$  there is a probability  $1 - \pi$  that research will show the idea is incorrect.
- At  $T$ , if we get there without the idea being shown incorrect, the price of the investment security will be  $Q_T = X$ .

## Calculating price, from $T$ backward in time

- At  $T - 1$  the price will be the expected value of the security at  $T$  (accounting for the chance the idea will be shown incorrect), discounted to  $T - 1$  by the market discount factor  $\phi$ , i.e.  $Q_{T-1} = \phi \cdot (\pi Q_T + (1 - \pi) \cdot 0) = \phi\pi X$ .
- continuing this calculation backward we get that  $Q_{T-s} = (\phi\pi)^s X$ , or equivalently  $Q_t = (\phi\pi)^{-t} Q_0$ , where  $Q_0 = (\phi\pi)^T X$ .
- If, say,  $T = 20$ ,  $\phi = .95$  and  $\pi = .9$ , the price rises by the factor  $(.95 \cdot .9)^{-1} = 1.17$  each period, unless it crashes. The chance that it won't ever crash, so the payoff of  $X$  at  $T = 20$  is obtained, is  $\pi^{20} = .9^{20} = .12$ , so it is unlikely that the investment will deliver anything.

- Accordingly, the price at  $t = 0$  is low:  $(.9 \cdot .95)^{20} X = .044 \cdot X$ , reflecting the combined effects of discounting over 20 periods at rate .95 and the probability that the payoff will actually occur.



## Peso problems

- Risk of rare disaster.
- The name comes from a situation where a government with a fixed exchange rate has to pay market interest rates on domestic currency (“peso”) debt well above those on, say, US Treasury bills, despite historical data showing that there is little risk of devaluation.
- If the risk is of a rare event — e.g. once in thirty or fifty years — its probability will not be estimated accurately from a historical record of less than a hundred years, in which it is likely to have occurred once, twice or not at all.
- Nonetheless if the event would have a large effect on returns, as would a sudden large devaluation of the peso, it could have a substantial effect on interest rates.

## Summary of the above

- All these examples — peso problem, growth rate news, and “waiting for bad news” — are cases where properly functioning asset markets could produce sudden drops in asset values following periods of high asset returns.
- These might look like asset market malfunctions, which we will discuss below.
- The point of looking at these examples: it is not easy to distinguish a “crisis” that is just markets functioning correctly from one that arises from market malfunction.

## “Rational” bubbles

- A security that never will pay any dividends, and everyone knows that.
- Its value will eventually drop to zero, and everyone knows that. Say the probability of its *not* crashing to zero is  $\pi$  each period.
- Its price today is  $Q_t$ . For someone (risk-neutral) thinking of buying it today to sell tomorrow, it must be that

$$Q_t = \phi\pi Q_{t+1}$$

- So investing makes sense so long as  $Q_{t+1}/Q_t > (\phi\pi)^{-1}$ .
- $Q$  can grow at faster than the inverse discount rate until the crash, even though everyone knows there will be a crash!

## Why “rational” is in scare quotes

- For this to work, it must be true that investors always think further price growth is possible.
- But if  $Q$  gets high enough, the value of the security must start to dominate the whole economy — buyers would realize that if they tried to use their wealth to buy consumption goods, this would be impossible — there aren't that many consumption goods in the world.
- So *before* this point is reached, people will see it coming and decide  $Q$  can't keep growing, so they won't buy the security.
- And so on back to the start — people should see this can't go on forever, and that should make them unwilling to hold the security at the start.

- But it sometimes looks as if investors do not look that far ahead.
- If you think there are a few fools who can't see as far ahead as you, so that you can surely sell before they realize what's going on, it's a good investment.
- It could even be that everyone thinks there are such fools, though there aren't any. That is, everyone is a fool for thinking they are smarter than some other market participants and can sell off the bubble asset before the crash.

## Ponzi schemes

- A “rational” bubble isn’t organized by anyone. It arises spontaneously.
- A Ponzi scheme is organized by someone. Usually it involves paying high returns to investors by using the money coming in from new investors to pay returns to old investors.
- In a rational bubble, though, we supposed that everyone, or at least most investors, knew that there would never be any dividends.
- A Ponzi scheme is instead a fraud, with investors not realizing that dividends are being funded entirely out of new investors share purchases.

- Like a bubble, a Ponzi scheme may be fed by people who see early high returns and extrapolate them into the future.
- And as with a bubble, there could be investors who understand that the scheme must crash, but believe that they can invest, then sell before the crash.
- US courts generally have held that investors who make money off a Ponzi scheme by investing, then withdrawing profits, must return the profits when the scheme crashes, in order partly to compensate those who lose their entire investments.