Asset Pricing and Term Structure

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This lecture is related to "Operation Twist"

- The Fed is trying to lower long term rates by buying long debt, selling short debt.
- The theory we are going to discuss suggests this will not work.

Asset pricing without uncertainty

- Price of one dollar delivered one year from now is ϕ. ϕ is the one-year **discount factor**.

- In a **competitive** asset market, we can buy or sell any amount of future dollars at this same unit price.

- Treasury bills are priced this way. The "discount basis" one year interest rate is $1 - \phi$.

- If we purchase one dollar's worth of this security today, we get $1/\phi$ dollars in one year.

- The “coupon basis” interest rate is $1/\phi - 1$. When ϕ is close to one, the coupon basis and discount basis rates are nearly the same.

- At high interest rates, the coupon basis rate is higher.

Multi-year rates

- Notation: ϕ_t^s is the s-period discount factor that prevails at time t: I can buy one dollar delivered at $t + s$ by paying ϕ_t^s at t. (The superscript s is not an exponent.)

- Suppose one dollar delivered two years from now can be obtained by paying now θ dollars.

- The two-year rate, annualized, discount-basis, is $1 - \sqrt{\theta}$.

- The idea is that if the one-year discount factor were the same value ϕ this year and next, we would need ϕ dollars at the beginning of next year to produce one dollar two years from now, and to get ϕ dollars at the beginning of next year we would need ϕ^2 dollars at the beginning of this year.

- So with a constant one-year rate, $\theta = \phi^2$, $\phi = \sqrt{\theta}$, and the annualized two-year rate is $1 - \phi = 1 - \sqrt{\theta}$.

- Generally, if θ is the discount rate at which one dollar s periods from now is sold, the s-period rate is $1 - \theta^{1/s}$. This works even for $s < 1$, e.g. $s = 1/12$ in the case of a monthly rate.

Mult-year rates and one-year rates

- Suppose one-year discount factors ϕ_{t+s}^1 that will prevail in the future are known at t.

- Then, since one can deliver a dollar in two years by buying at t enough one-year security so that we can at $t+1$ buy another one-year security that delivers a dollar at $t+2$, it must be that

$$\phi_t^2 = \phi_t^1 \phi_{t+1}^1 \,.$$

- More generally, when the future one-year discount factors are known,

$$\phi_t^s = \prod_{v=0}^{s-1} \phi_{t+v}^1 \; .$$

- If we take logs, $\log \phi_t^s = \sum_{v=0}^{s-1} \log \phi_{t+v}^1$.

Multi-year rates as averages of future one-year rates

- Recall that the discount-basis s-period net interest rate is $1 - (\phi_t^s)^{1/s}$. When the net interest rate is only a few percent, i.e. $(\phi_t^s)^{1/s}$ is close to one, the net rate r_t^s is close to

$$-\log\big((\phi_t^s)^{1/s}\big) = -\frac{1}{s}\log\phi_t^s = -\frac{1}{s}\sum_{v=0}^{s-1}\log\phi_{t+v}^1 \doteq \frac{1}{s}\sum_{v=0}^{s-1} r_{t+v}^1$$

Pricing anything from current and future one-year rates

- Suppose that at date t we know ϕ_t^s, the s-period discount factor (or $1 - (\phi_t^s)^{1/s}$, the s-period interest rate) for every s.

- Then suppose we had to determine the price of a security that pays a known dividend δ_{t+s} at each date $t + s$ for $s = 1, \ldots, T$.

- This security is equivalent to a collection of s-period discount bonds, so we can see its price has to be

$$\sum_{s=1}^{T} \phi_t^s \delta_{t+s} \, .$$

- Converting a stream of future payments to a current price in this way is known as finding the **present value** of the payment stream.

The term structure

- The term structure of interest rates is the set of s-period rates $\{r_t^s\}$ prevailing at a given date t.
- It is often displayed in a plot of r_t^s against s, called a **yield curve**.
- In the absence of uncertainty about the future, we could deduce all the future one-year rates from knowledge of the term structure.
- Even with uncertainty, we can interpret the term structure as suggestive of beliefs about future short-term rates: when the plot is sharply rising, interest rates are expected to be rising, when the plot is falling, they are expected to be falling.

- Uncertainty does make a difference. For example, because of uncertainty usually short rates are lower than long rates. (We'll see why later.)

- When market beliefs that current short rates are unusually high are strong enough, though, the yield curve slopes downward. This is called an "inverted yield curve".

Pricing long term bonds

- A typical long term bond with maturity T and face value B pays a **coupon** $r \cdot B$ every year until T, when it returns the **principal** B.

- If all future one-year rates are equal to r and known, then pricing the bond by discounting its coupons and principal payment makes it worth B at the date of issue.

- However, interest rates change, and this makes the value of the bond change.

Pricing consols

- A consol is an infinite-maturity $(T = \infty)$ bond. If its face value is B and its interest rate at issue was r, it just pays rB every year forever.

- If the term structure is flat at t, with all rates equal to r_t, with $r_t \neq r$ possibly, then there is a constant one-period discount factor $\phi = 1/(1 + r_t)$ and the consol is worth

$$\sum_{s=1}^{\infty} \phi^s rB = \frac{\phi rB}{1 - \phi} = \frac{rB}{r_t} \, .$$

- The income stream from the consol is riskless, but the price will vary sharply as interest rates vary, so for someone who might have to sell the consol at a future date, it is a risky investment.

A general coupon bond

Suppose the bond has the coupon rate r, maturity T, and a principal value B, and that the current interest rate is r_t and expected to remain at r_t in the future. Let $\phi = 1/(1+r_t)$ be the discount factor. Then the bond price today is (again under our assumption of perfect foresight)

$$
\begin{aligned}
Q_t = \phi^T B + \sum_{s=0}^{T-1} rB\phi^s &= B\left(\phi^T + r\frac{\phi(1-\phi^T)}{1-\phi}\right) \\
&= B\left((1+r_t)^{-T}\left(1-\frac{r}{r_t}\right)+\frac{r}{r_t}\right) .
\end{aligned}
$$

This equation follows from the formula for the sum of a geometric series:

$$\sum_{s=0}^{T-1} \rho^s = \frac{1-\rho^T}{1-\rho}\,,$$

together with some algebra and use of the $\phi = 1/(1+r_t)$ definition. Note that with $T = \infty$, so long as $r_t > 0$, this is just the formula for the price of a consol that we derived above. Also, with $r_t = r$, $Q_t = B$.

Real vs. nominal returns, TIPS

- US Treasury securities are for practical purposes risk free in dollar terms.
- But for, say, someone saving for retirement 20 years from now, there is nonetheless a big risk, from inflation.
- What such a person cares about is not the dollar value of the investment in 20 years, but rather how much the investment will buy.
- Unexpectedly high inflation over the next 20 years could make his investment inadequate to support him in retirement.
- Treasury Inflation-Protected Securities (TIPS) offer protection agains inflation buy adjusting returns to be higher when inflation is high.

- The gap between the yield on ordinary treasury securities and TIPS is often used as a measure of “market expectations” of inflation.

The real interest rate

- Ex post gross: $R_t P_t / P_{t+1}$; Ex post net: $r_t - \pi_{t+1}$, where π_t is the inflation rate.

- Ex ante gross: $E_t[R_t P_t / P_{t+1}]$; Ex ante net: $r_t - E_t \pi_{t+1}$.

Liquidity and risk

- In reality, future payments and interest rates are not known with certainty. Asset returns are therefore subject to **risk**.

- People prefer riskless to risky investments, so risky assets have lower prices.

- A related, but distinct, concept is **liquidity**. A liquid asset is easily sold quickly. People dislike illiquid assets, so they have lower prices.

- A long-term treasury bond is risky, because its value is sensitive to future fluctuations in interest rates, yet it is highly liquid, because there are active, “thick” markets in treasury bonds.

Examples

- A bank loan to a local small business that the bank knows very well may be extremely low-risk, but because any buyer of the loan would have to do her own investigation of the borrower's credit-worthiness, the loan is illiquid.

- Liquidity has to do with how close the asset's market is to the competitive ideal, in which arbitrarily large amounts can be bought or sold at arbitrary times without affecting the market price at those times.

- Risk has to do with uncertainty about future incomes from and values of an asset.

- If it is possible that the asset might need to be sold quickly, illiquidity can create risk.

- A risky asset often is one where buyers feel a need to do more "due diligence" before buying, and hence may be less liquid.

Risk, liquidity and the term structure

- The normally upwardly sloped shape of the yield curve arises from risk and liquidity considerations.

- Even though long treasury bonds are highly liquid, the shortest bonds are the most liquid. Since they expire frequently, they are constantly bought and sold in large amounts, making the market "thick" — able to absorb large sales with little effect on the price.

Diversifiable and non-diversifiable risk

- The "risk premium" is the extra return that an asset must pay to compensate for its riskiness.
- It is not determined by the uncertainty in the return itself. It is determined by how much **undiversifiable** risk there is in the return.
- If there are many stocks available, each individually quite risky, an investor can split up her investment into small portions, investing each portion in one of the risky stocks.
- If the the individual stock risks are not related, the full investment portfolio will be much less risky than it would be if it all went into one of the stocks. This is diversifiable risk.

- But if instead the main risk to all the stocks depended on the state of the economy, with all declining together in a recession or rising together in a boom, The portfolio of many small investments would be about as risky as investing in one of the stocks. This is undiversifiable risk.

The risk premium

- If market participants believe that the risk on a security is diversifiable — that it can be combined with other risky securities to deliver a much less risky portfolio — they will not require a much higher return (i.e. much lower price) to invest in it, even if its return is by itself quite uncertain.

- What generates a large risk premium is uncertainty about returns where the uncertainty is common across many investments, so that it cannot be averaged away.

- Risk premia are high or low based on what market participants *believe* about the amount of uncertainty and whether it is diversifiable. These beliefs can be mistaken.

Example: securitized mortgages

- Before the recent crisis, it had become common practice to securitize mortgages.
- Banks used to lend money to house-buyers and retain the mortgage as an asset on their balance sheets.
- With securitization, they lent the money, but then sold the mortgage to a firm that collected many mortgages, then sold shares in the collection to investors.
- Mortgages are risky, because borrowers sometimes default.
- Historically, defaults tended to occur in a few spots around the country where housing markets were troubled, but never all at once across the country — they appeared to present diversifiable risk.

Securitized mortgages, continued

- In fact, it turned out to be quite possible for housing markets to decline nearly in sync across the country.
- And the nature of the innovations in mortgage finance had made it more likely that defaults would occur in sync rather than independently.
- Robert Merton's lecture yesterday was about this point.
- For example, for a number of reasons there was increased use of ARM's, adjustable rate mortgages, and the increase in this type of mortgage was fairly sudden. These had low initial interest rates that would increase at some known future date.
- If the economy soured, so that many of these borrowers couldn't handle increased monthly payments, many would default at once.

Securitized mortgages, continued

- The securitized mortgages were priced and traded as if they involved diversifiable risk, as historical data suggested.

- A few people realized that the risks were likely not in fact diversified, but there were not enough of them to force an early repricing of the risk.

- Instead, they made a lot of money when the values of the securities crashed.

- See `The Big Short` by Michael Lewis.

Slicing and dicing risk

- In fact, markets innovators went beyond just bundling mortgages to reduce average risk.
- They bundled mortgages and split them into “tranches”.
- All defaults were attributed to the bottom tranche, until it was “full”, then they were allocated to the next tranche up, etc.
- If there were, say, 4 tranches, the top tranche saw no default risk at all until 75% of the mortgages had defaulted.
- If, say, 10% of mortgages defaulted on average, the risk of any defaults in the top tranche was essentially zero.

- Such tranches were given AAA ratings by ratings agencies.

- But if there was a substantial undiversifiable risk, the chance of losses in the top tranche was much higher than would be suggested by assuming independence.