EXERCISE ON ASSET MARKETS, DUE IN CLASS THURSDAY, OCTOBER 13, AND PRECEPT DISCUSSION QUESTIONS

(1) **CDO's and securitization**
Financial institutions and regulators often summarize the riskiness of a portfolio or an investment by reporting a "value at risk", or VaR. This will be something like "The one percent probability VaR is three percent", meaning that losses of three percent or more have a probability one percent.

Consider an investment that costs 1.0 today and delivers 1.06 or .96 tomorrow, with equal probability on the two outcomes. We could say this security has a 50 percent probability VaR of four percent. Suppose we combined this mortgage with another mortgage with the same costs and returns, but with the randomness in returns independent of the first mortgage. If we sold two shares in this bundle of assets, at a price of 1.0 per share, there would be three possible percentage returns: 1.06 (if both turned out well), 1.01 (if one delivered 1.06 and the other .96) and .96. The probabilities of the three outcomes would be .25, .5, and .25, respectively. We could say the 25% VaR is four percent for this security, so it is clearly safer by this measure than the individual securities.

(a) Suppose we bundle four independent securities with the same distribution of returns as the individual securities above, and sell four shares at a price of 1.0. In this case 0, 1, 2,3, or 4 of the securities could deliver the 1.06 return. In case your binomial distribution chops are a bit rusty, the probabilities of these outcomes are .0625, .25, .375, .25 and .0625, respectively. Calculate the VaR values corresponding to the two lowest returns of these five possible returns and show how they would be quoted as a VaR (i.e., x% probability VaR of y%).
The two lowest values occur when all four, or three of the four, deliver .96. In the former case, which has probability .0625, the return is .96, so we can say the 6.25% probability VaR is 4%. In the latter case the return is -1.5% and the probability is .25, so the 25% VaR is 1.5%.

(b) Now suppose we split our bundle of four securities into two "tranches" to create "collateralized debt obligations" (CDO's). The upper tranche will pay the returns of the best two of the four securities in the bundle, and the lower tranche the returns of the worst two of the four. Note that with the individual securities selling at 1.0, with expected return

$\frac{1}{2}(1.06 + .96) = 1.01)$, we can (ignoring risk premia) take the discount factor to be $1/1.01$ (so the current price of 1.0 is the discounted expected return). Determine the expected return on the upper and lower tranches and their prices with discounting by this $1/1.01$ factor. Also quote a VaR for each of the two.

The upper tranche pays 1.06 when 2, 3, or 4 of the securities pay 1.06, and that occurs with probability .6875. When only one pays 1.06, the payout is 1.01, and that has probability .25. And when none pays 1.06, the payout is .96, and that occurs with probability .0625. So the expected payout is $.6875 \times 1.06 + .25 \times 1.01 + .0625 * .96 = 1.04125$. That means that the current price is $1.04125/1.01 = 1.0309$. The VaR for this security, then — calculated as a percentage of the investment required, is a probability .25 VaR of $(1.0309 - 1.01)/1.0309$, or 2.03%. Or one could quote a probability .0625 VaR of $(1.0309 - .96)/1.0309$, or 6.9%. The lower tranche has return .96 if 0, 1, or 2 of the securities pay 1.06, which has probability .6875. It pays 1.01 if three pay 1.06, which has probability .25, and 1.06 if all four pay 1.06, which has probability .0625. Thus its expected payout is $.6875 \times .96 + .25 \times 1.01 + .0625 \times 1.06 = .97875$ and therefore a current price of .9691. It therefore has 68.75% VaR of $(.9691 - .96)/.9691$, or 0.93%.

(c) Is the upper tranche CDO less risky than the simple bundling of the four securities into one with equal shares? (Warning: "Yes" or "no" may both be wrong answers.)

The probability .25 VaR is 1.5% for the bundle, and 2.03% for the tranche. The probability .0625 VaR is 4% for the bundle, 6.9% for the tranche. So the risk of loss is greater for the tranche at these probability levels. On the other hand the upper tranche delivers a gross rate of return of $1.06/1.0309$, or 2.82% net, with 68.75% probability, whereas the bundle can guarantee only a return greater than or equal to 1% with that level of probability. This is a general characteristic of tranched securities — they deliver a good return with higher probability than a corresponding bundle, but they suffer *larger* percentage losses when outcomes are very bad. If we had considered a bundle of many more securities, and considered a tranche of a smaller upper fraction of the returns, the probability of no loss could have been made much larger, but it would remain true that the rates of return in the unlikely cases where the top tranche suffered losses would be worse than for the bundled securities.

Discussion questions

(i) In the US, when a Ponzi scheme ends, those who are left with worthless investments can legally recover money from investors who invested in the Ponzi scheme earlier and made net profits by withdrawing funds before the scheme ended. An alternate rule might be simply that bygones are bygones, the perpetrator of the scheme can go to jail and be sued, but investors who happened to make a profit at early stages get to keep their profits. Could the choice between these legal rules make a difference to how easy it is to sustain a Ponzi scheme? Why?

(ii) Could the "internet bubble", in which internet-related stocks rose in value, then crashed, have represented properly functioning asset markets and rational investor behavior? What about the "housing bubble"?

(iii) In class we discussed an example economy with "farmers" and "bakers", in which using gold as money saved people search and information costs required to make individual bilateral contracts for delivery of grain to bakers and bread to farmers. We also discussed how a bank could create more convenient money by storing gold and issuing gold certificates that people could use as money. While this story seems natural, and is even something like one way banks did arise in some places and times, it leaves a question: Why couldn't everyone just issue gold certificates for whatever gold they had? This would serve the same purpose as having the goldsmith/bank store the gold and issue certificates. Is it a less plausible story? Why?