Linear Rational Expectations Model Exercise

Consider the following specialization of the permanent income model we have considered, in which U is quadratic and r is constant.

$$\boxed{\max_{\{C_t,A_t\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} b^t\left(C_t - \tfrac{1}{2}C_t^2\right)\right]} \tag{1}$$

subject to

$$\text{l}: \quad A_t \le (1+r)A_{t-1} - C_t + Y_t \tag{2}$$

$$\text{m} \quad A_t \ge 0 \tag{3}$$

$$Y_t = .25 + 1.5Y_{t-1} - .75Y_{t-2} + z_t \ , \tag{4}$$

where we assume $E_t z_{t+1} = 0$, all t. All the constraints hold for all $t \ge 0$.

Note that we are giving the constraints as inequalities, and that the Lagrange multipliers associated with constraints are zero at times and states of the world when the constraints are not binding. Note also that there is no Lagrange multiplier for (4), because it contains no decision variables; it simply restricts the behavior of the exogenous variables. However it will be part of the equation system defining the solution (and therefore will have to be cast into first-order form.) The FOC's for this system, together with the constraints, form a linear equation system, so long as the inequalities are either always binding or always not binding. We usually expect to find solutions in which (2) binds and (3) does not, but solutions of the opposite form need sometimes to be considered. There are cases in which the binding constraint varies from time to time, stochastically, but these will not fit into the framework of a simple linear equation system – the equation system itself varies stochastically over time in these cases.

i) Derive the first order conditions for this problem, assuming that (3) never binds and (2) always binds.

ii) Derive the first order conditions assuming that (2) never binds and (3) always binds.

iii) For each of the cases above, display the values of the matrices Γ_0, Γ_1, Ψ, and Π from the canonical form of the stochastic difference equations given in the notes on linear rational expectations models, when this problem is cast in to the canonical form.

iv) Using the methods explained in the Jordan Decomposition section of *Solving Linear Rational Expectations Models*, determine whether either of the two cases in (i)-(ii) above generate an optimum, and whether it is unique. Also find the relationship between expectational errors h and exogenous disturbances z in the solution, if any. Note that the answers may depend on the values of b and r, so that your answer may need to be divided up across a number of possible cases. Finally, note that at satiation, i.e. $C_t = 1$, all t, the FOC's may not apply, because the set of points preferred to this C path has empty interior, yet this possible C path may be interesting to consider.

v) Discuss what limits on the rates of growth of C and A are implied by the transversality conditions and feasibility constraints for this model.