Final Exam

The exam contains five questions. Answer all five questions. Points for each question are indicated beside the question. If you spend one minute per point you will use two hours and 35 minutes. You will have 3 hours to finish the exam.

1. (45 points) Consider an economy in which identical agents maximize

$$E\left[\sum_{t=0}^{\infty}\beta^t\cdot\left(\log C_t-\gamma L_t^2\right)\right] \tag{1}$$

subject to

$$C_t+\frac{B_t}{P_t}+\tau_t\le R_{t-1}\frac{B_{t-1}}{P_t}+A_tL_t^{\alpha}\,, \tag{2}$$

$$B_t\ge 0\,, \tag{3}$$

choosing time paths for B, C and L, taking A, P and R as given. As usual, agents know all variables dated t and earlier when making choices dated t, but do not know the values of variables dated later than t when making choices at t.

The government budget constraint is

$$\frac{B_t}{P_t}+\tau_t=R_{t-1}\frac{B_{t-1}}{P_t}+G_t\,, \tag{4}$$

where G is real government purchases. Government policy sets

$$R_t=\overline{R}=\beta^{-1} \tag{5}$$

$$\frac{\tau_t}{C_t}=\overline{\tau} \tag{6}$$

$$\frac{G_t}{C_t}=\overline{G}\quad. \tag{7}$$

Note that the relationships in (6) and (7) characterize the behavior of the economy-wide average of taxes and spending. Individuals as usual treat taxes as lump-sum, i.e. as invariant to their individual consumption choices. We assume that the economy starts up with a positive B.

a) Find the first-order conditions for a solution to the consumer's optimization problem.

b) Show that the economy has a competitive equilibrium in which P_tC_t remains constant.

c) Show that this equilibrium is unique. [It is likely to be helpful to divide the government budget constraint (4) by C_t and to take E_{t-1} of it, using the FOC with respect to B.]

d) Find the the equilibrium paths of P, C and L as functions of constants and the disturbance A_t.

e) What are the effects on the paths of P, C and L of an unanticipated one-time, permanent increase in $\overline{G}$ accompanied by an equal increase in $\overline{\tau}$ (a balanced-budget fiscal expansion)?

2. (30 points) Consider the following equation system, meant to correspond to a simple overlapping contracts setup, simplified so that real activity is constant.

Fisher relation: $$r_t = \bar{r} + E_t p_{t+1} - p_t \tag{8}$$

Money demand: $$\overline{m} = \bar{y} + p_t - \theta r_t + \varepsilon_t \tag{9}$$

Wage adjustment: $$w_t = .5 w_{t-1} + .25 \cdot \left(p_t + E_t p_{t+1}\right) + \xi_t \tag{10}$$

The variables with bars over them are constants, with m constant because policy sets it to be constant. $\bar{r}$ and $\bar{y}$ are determined by the technology. All the variables are interpreted as logarithms, and $E_t \varepsilon_{t+1} = E_t \xi_{t+1} = 0$. Show that if we rule out rapidly explosive solutions, this model has a uniquely determined price level at each date. Find a formula for p_t as a function of constants and the shocks ε and ξ. [It is probably easiest to begin by using (9) to eliminate r from the system, so it becomes a 2-dimensional model.]

3. (15 points) Consider an economy consisting of a large and equal number (each of mass 1) of two types (called odds and evens) of infinitely lived agents. There is a single non-storable type of consumption good. 'Odd' agents receive the endowment stream $\{y_t^o\}_{t=0}^{\infty}$, while 'even' agents receive the endowment stream $\{y_t^e\}_{t=0}^{\infty}$. The endowment streams are given by:

$$y_t^o = \begin{cases} 1 & \text{if } t \text{ is odd} \\ 0 & \text{if } t \text{ is even} \end{cases}$$

and

$$y_t^e = \begin{cases} 0 & \text{if } t \text{ is odd} \\ 1 & \text{if } t \text{ is even} \end{cases}$$

Agents of type i wish to maximize:

$$\sum_{t=0}^{\infty} \beta^t \ln c_t^i, \quad i = \text{odd, even}, \quad \beta \in (0,1)$$

where c_t^i is the time t consumption of the single good by an agent of type i.

Assume agents can borrow from and lend to other agents using one period riskfree loans.

(a) Define an *equilibrium with one-period bonds* for this economy.

(b) Compute the prices and quantities for an equilibrium in this economy. [Hint: Recall we did this in class. You may use the "guess and verify" method.]

4. (30 points) Consider an economy similar to one described in the previous problem. But in this economy, at time zero, 'odd' agents own a stock which pays a dividend, $\delta_o(t)$, equal to one each odd period: That is,

$$\delta_o(t) = \begin{cases} 1 & \text{if } t \text{ is odd} \\ 0 & \text{if } t \text{ is even} \end{cases}$$

Similarly at time zero, 'even' agents own a stock which pays a dividend, $\delta_e(t)$, equal to one each even period: That is,

$$\delta_e(t) = \begin{cases} 0 & \text{if } t \text{ is odd} \\ 1 & \text{if } t \text{ is even} \end{cases}$$

As in the previous problem, agents of type i wish to maximize:

$$\sum_{t=0}^{\infty} \beta^t \ln c_t^i, \quad i = \text{odd, even}, \quad \beta \in (0,1)$$

where c_t^i is the time t consumption of the single good by an agent of type i. Assume all borrowing and lending is prohibited, but agents can trade in stocks.

(a) Write down each agents' maximization problem including the budget constraint(s).

(b) Define an equilibrium for this economy with a stock market.

(c) Compute the consumption allocation and stock price paths for this equilibrium.

(d) In class, we examined endowment economies with bonds only and with traded equity. Those models differed from the ones in problems 1 and 2 of this exam only in that endowments were random rather than deterministic. In the models with random endowments, allocations differed according to whether bonds or equities were traded. Compare the results from that setting with random endowments to what you have found here with determinstic endowments and explain any differences you note.

5. (30 points) Consider an economy with a continuum of *ex ante* identical households (with mass 1) each of whom evaluates consumption streams according to

$$E \sum_{t=0}^{\infty} \beta^t u(c_t).$$

Let c_t denote the consumption of an agent at time t.

Every period each household draws an employment opportunity s_t. Assume s_t can take on one of two values: employed or unemployed. Assume s_t evolves according to a two state Markov chain with a transition matrix $\mathcal{P}$ where:

$$\mathcal{P}(s, s') = \text{Prob}(s_{t_1} = s' | s_t = s) \quad \text{for} \;\; s, s' \in \{\text{employed,unemployed}\}.$$

Each period the household receives $y(s_t)$ in labor income. This household specific randomness is distributed identically and independently across households. In other words, there is no aggregate uncertainty in this economy.

The only asset the households can hold is an unbacked currency that is issued by the government. Let $\tilde{m}_{t-1}$ denote the household's nominal money balances carried from period $t-1$ to period t. Households cannot issue money; so $\tilde{m}_t \geq 0$. Let $\tilde{M}_{t-1}$ denote the per capita nominal money supply at time $t-1$. New money is injected into this economy through lump-sum transfers $\tau \tilde{M}_{t-1}$. So the government budget constraint is:

$$\tilde{M}_t = (1+\tau)\tilde{M}_{t-1}.$$

Thus the household's constraints are:

$$c_t + q_t \tilde{m}_t = q_t \tilde{m}_{t-1} + y(s_t) + \tau q_t \tilde{M}_{t-1}$$
$$\tilde{m}_t \geq 0,$$

where q_t is equal to value of money in terms of goods.

Let π_t denote the inflation rate, so

$$\frac{q_t}{q_{t-1}} \equiv \frac{1}{1+\pi_t},$$

and let m_t denote the household's real money balances, so

$$m_t = q_t \tilde{m}_t.$$

Then we can write the household's constraints as:

$$\begin{aligned} c_t + m_t = \frac{1}{1+\pi_t} m_{t-1} + y(s_t) + \frac{\tau}{1+\pi_t} M_{t-1} \\ m_t \geq 0, \end{aligned}$$

where $M_t = q_t \tilde{M}_t$.

(a) Write down the household's Bellman equation.

(b) Define a stationary equilibrium for this economy.

(c) Describe an algorithm to compute this stationary equilibrium. (Do not try to solve for the stationary equilibrium. It would take more computing power than you can access at the moment.)