Common Mistakes on Final Exam Questions 1 and 2

Overall, performance was very good on problem 1, disappointing on problem 2. The average score was about 2/3 of possible credit on 1, 1/3 of possible credit on 2.

On 1, a common difficulty was that people would correctly calculate the unique equilibrium value for $B/(PC)$, and have the result that *PC* is constant, but not see how to turn that into an expression for equilibrium *P*, despite having expressions for C and L as functions of the exogenous shocks. The point to remember: In these fiscal models of the price level, the price level will generally depend on the initial value of *B* (or of $B+M$ in a model with money). One gets the initial price level from

$$\frac{B_0}{P_0 C_0} = \bar{b} \ , \ \therefore P_0 = \frac{\bar{b}\, C_0}{B_0} \ . \qquad [1]$$

This pins down $P_0 C_0$, and with $P_t C_t$ constant and C_t determined as a function of disturbances, we then know the behavior of P_t for all *t*. Though this is fairly simple, it does involve recognizing that we can't use steady-state equations alone to pin down *P*, because there is no unique steady-state *B*.

On 2, scores were uncorrelated with performance on the rest of the exam. I think this is because people who had prepared thoroughly and were ready to display their understanding of how to use matrix methods on linear rational expectations models plunged ahead without thinking carefully enough about how to simplify the problem, while people who were less sure of the matrix methods often tried successfully to see a way to avoid the matrix methods altogether. Probably there were too few class exercises providing practice in solving small linear RE models, as opposed to practice in setting up models for the computer to solve.

I think there was only one exam where a matrix approach was set up without mistakenly altering the timing of error terms. To use the methods of the "Linear Rational Expectations Models" notes, it is essential that one keep the model in the form where for the artificial error vector η_t, $E_t \eta_{t+1} = 0$. It is possible to proceed without the assumption that for exogenous shocks ε_t, $E_t \varepsilon_{t+1} = 0$, but doing so requires using forward-solution methods that we didn't discuss in class for the matrix case and that are not discussed in the Jordan decomposition section of the notes that was emphasized. It is also necessary to maintain notation in which all variables dated *t* are known at *t*. This means that it is OK to introduce a dummy variable $z_t = w_{t-1}$ in the problem of question 2, but not to introduce a dummy variable $z_t = p_{t+1}$. (The posted suggested answer uses a dummy $z_t = E_t p_{t+1}$, which is OK because $E_t p_{t+1}$ is known at *t*.)

The consequences of messing up the timing of variables and errors is not to change the roots or the analysis of stability. It is to change the way that error terms enter the stability conditions. Those people who used matrix methods and reached a conclusion, in I think every case concluded that *p* was constant, when it is in fact dependent on the money demand shock.

The other common type of error was clumsy or rusty matrix algebra. Quite a few exams calculated a canonical $\Gamma_0^{-1}\Gamma_1$ of triangular form without realizing that in a triangular matrix the roots are simply the diagonal elements of the matrix. Also, a major appeal of the Jordan decomposition approach is that it does not require that one compute the entire $P\Lambda P^{-1}$ decomposition in order to find the stability condition, and most exams showed no recognition of this. If there is just one unstable root, as in this question's model, the stability condition is obtained by multiplying the system through by the left eigenvector corresponding to the unstable root. Finding this one eigenvector is much easier in general (and in this problem very much easier, because it is a unit vector) than finding the whole of P and P^{-1}. To be specific, once the system has been multiplied through by Γ_0^{-1} to reach the form

$$y_t = Ay_{t-1} + \Psi^*\varepsilon_t + \Pi^*\eta_t + C^* \ , \tag{2}$$

with $A = P\Lambda P^{-1}$, we can multiply on the left by any row *j* of P^{-1} to obtain

$$P^{j\bullet}y_t = \lambda_j P^{j\bullet}y_{t-1} + P^{j\bullet}\Psi^*\varepsilon_t + P^{j\bullet}\Pi^*\eta_t + P^{j\bullet}C^* \ . \tag{3}$$

Stability then requires, when $|\lambda_j| > 1$ by too much, that

$$\begin{gathered} P^{j\bullet}y(t) = \left(1-\lambda_j\right)^{-1}P^{j\bullet}C^* \\ P^{j\bullet}\left(\Psi^*\varepsilon_t + \Pi^*\eta_t\right) = 0 \end{gathered} \ . \tag{4}$$

To find the single left eigenvector corresponding to the single root λ_j, one solves the equation

$$xA = \lambda_j x \ . \tag{5}$$

This of course requires normalizing *x* somehow, usually by setting one element to 1.