Econ. 511b Spring 1997 C. Sims

Asset Pricing Exercise

The model discussed in class with a single asset paying a stochastic yield is defined as follows. There are two agents, $i = 1,2$, each maximizing an objective function of the form

$$E\left[\sum_{t=0}^{\infty} \beta^t U_i(C_{it})\right] \tag{1}$$

subject to the constraints, respectively,

$$C_{1t} + Q_t S_t = Y_{1t} + (Q_t + \delta_t) S_{t-1} \tag{2}$$

$$C_{2t} - Q_t S_t = Y_{2t} - (Q_t + \delta_t) S_{t-1} \tag{3}$$

In order to have a competitive equilibrium solution, there must be some sort of constraint that makes agents see borrowing arbitrarily large amounts as impossible. A simple way to do that is to say that the two agents see the constraints, respectively,

$$Q_t S_t \geq -H \tag{4}$$

$$-Q_t S_t \geq -H \ . \tag{5}$$

Assume Y_1 and Y_2 are identically distributed and independent of each other and across time. We take their common mean to be $\bar{Y}$. We will make specific assumptions about the behavior of δ_t, but we will always use $\bar{\delta}$ to refer to the constant mean of δ.

i) Find the deterministic steady state value of Q as a function of $\bar{\delta}$ and β.

ii) Assume $\delta_t = Y_{1t} - Y_{2t}$. Show that an economy in which $S_t = -.5$ for all t can be in competitive equilibrium, and that the equilibrium has C_{it} matching a planner's allocation with equal weights on the two agents. You will have to show that there is a price process Q_t for the security that makes this equilibrium work.

iii) Show that the equilibrium satisfies the following weak uniqueness condition: At every t, $S_t = -.5$ is the unique value of S at which both agent's FOC's are satisfied, given the stochastic process for prices that you have determined. (In other words, you are not asked to show that there could not be another equilibrium with different prices. Only that at this equilibrium's prices, there is a unique pair optimal choices of S at each date for each agent.