Mid-Term Exam

You have 90 minutes to complete the exam. The questions have suggested numbers of minutes beside them, which add up to 80, leaving you 10 minutes of slack. Do not spend disproportionate time on any one question unless you have answered the others.

1. (20 minutes) Consider the model we have examined in class, with two agents i=1,2, each of whom maximizes

$$E\left[\sum_{t=0}^{\infty}\beta^t\frac{C_i(t)^{1-\gamma}}{1-\gamma}\right] \tag{1}$$

subject to

$$C_1(t)+\sum_{j=1}^{2}Q_j(t)S_j(t)=Y_1(t)+\sum_{j=1}^{2}\left(Q_j(t)+\delta_j(t)\right)S_j(t-1) \tag{2}$$

$$Q_j(t)S_j(t)\geq -H \quad \text{(agent 1)} \tag{3}$$

$$C_2(t)-\sum_{j=1}^{2}Q_j(t)S_j(t)=Y_2(t)-\sum_{j=1}^{2}\left(Q_j(t)+\delta_j(t)\right)S_j(t-1) \tag{4}$$

$$Q_j(t)S_j(t)\leq H \quad \text{(agent 2)}\ . \tag{5}$$

Agents choose C and S sequences subject to the usual requirement that choices of variables dated t must be based on knowledge of other variables dated t and earlier only. The two Y sequences are both i.i.d. across time, independent of each other, and have the same distribution.

Show that if the first asset has dividend payout $\delta_1(t)=\overline{Y}(t)=\left(Y_1(t)+Y_2(t)\right)/2$, if the economy is in a competitive equilibrium that reproduces the complete-markets solution in which $C_1(t)=C_2(t)=\overline{Y}(t)$ for all t, and if $E\left[\overline{Y}(t)^{1-\gamma}\right]=1$, then the equilibrium price of the first asset behaves according to $Q_1(t)=\phi\,\overline{Y}_t^{\gamma}$, where ϕ is a constant. Find ϕ as a function of β.

2. (20 minutes) Consider the standard permanent income model, in which the agent maximizes

$$E\left[\sum_{t=0}^{\infty}\beta^t\left(C_t-\tfrac{1}{2}C_t^2\right)\right] \tag{6}$$

subject to

$$W_t=(1+r)(W_{t-1}-C_{t-1})+Y_t\,, \tag{7}$$

$$W_t\beta^{-.5t}\to 0 \tag{8}$$

where Y is i.i.d. with positive mean.

This problem, as we have seen, leads, when $\beta \cdot (1+r) = 1$, to a decision rule of the form

$$C_t = \left(\beta^{-1} - 1\right) W_t \,. \tag{9}$$

We have also discussed a variant of this model, in which the equality in (7) is replaced by an inequality and (8) is replaced by the constraint

$$C_t \le W_t \,. \tag{10}$$

Using the model's first order conditions, explain why in this variant model C is lower than implied by (9) when W is either very low or very high.

3. (20 minutes) Consider the deterministic optimal growth model with preferences

$$\sum_{t=0}^{\infty} \beta^t U(C_t) \,, \tag{11}$$

where $0 < \beta < 1$ and $U(C_t) = \ln(C_t)$. The technology is

$$K_{t+1} = (1+\alpha)(K_t - C_t) \,, \tag{12}$$

where $\alpha > 0$.

a) Write the Bellman equation satisfied by the value function for this optimization problem. Show that a value function of the form $V(K) = A + D\ln(K)$ satisfies the Bellman equation. Find the values of A and D as functions of β and α.

b) Find the optimal policy function and the optimal growth rate of the capital stock.

4. (20 minutes) Consider the situation of an individual worker in an economy in which there are two types of jobs: low wage (w_1) jobs and high wage ($w_2 > w_1$) jobs. Low wage jobs are always available. High wage jobs are sometime available. We suppose the worker maximizes

$$E\left[\sum_{t=0}^{\infty} \beta^t w_t\right]. \tag{13}$$

If the worker has a low wage job today, he can keep this job forever, or he can search for a high wage job. If he searches for a high wage job, he earns nothing today, but may find a high wage job beginning next period, with probability θ. If the worker instead has a high wage job today, he earns w_2 today, but with probability λ loses the job and moves to a low wage job next period.

a) Display this worker's Bellman equation. [Hint: Get separate expressions for $V(w_1)$ and $V(w_2)$.]

b) For what values of w_1/w_2 (in terms of the parameters β, λ, and θ) will the worker with a low wage job always choose to search rather than work at w_1?