Econ. 511b Spring 1998 C. Sims

Solving a Linearized Model

I. The model to solve

In this exercise, you will linearize a specific version of the neoclassical growth model that does not have an analytic solution, and you will use the computer to get a solution that is valid in a neighborhood of the model's steady state. The same thing is done in the notes on linear approximation of the growth model that are posted on the web site, so those notes may be helpful to you. The notes deal with different functional forms for the technology, however, and do a lot of algebra aimed at getting analytic insight into the model. Here you will let the computer do more of the work.

The model is

$$\max_{\{C_t, I_t, K_t\}_{t=0}^{\infty}} E\left[\sum_{t=0}^{\infty} \beta^t \cdot \left(1 - e^{-\gamma C_t}\right)\right] \tag{1}$$

subject to

$$C_t + I_t \cdot \left(1 + \phi \frac{I_t}{K_{t-1}}\right) \le A_t K_{t-1}^{\alpha},\ t = 0, \dots, \infty, \tag{2}$$

$$K_t = \delta K_{t-1} + I_t \tag{3}$$

$$C_t \ge 0,\ K_t \ge 0 \text{ , all } t. \tag{4}$$

The single-period utility function has the form that is known as "constant absolute risk aversion". The technology defined by (2) has "adjustment costs" in capital. That is, the marginal rate of transformation between consumption and investment is not constant. The technology is linear homogeneous: scaling a $\{C, I, K\}$ that satisfies (2) by a common factor will leave (2) still satisfied.

i) Prove that the objective function defined by (1) is concave over the linear space of sequences of positive numbers $\{C_t\}_{t=0}^{\infty}$.

ii) Prove that the feasible set of three-dimensional sequences $\{C_t, I_t, K_t\}_{t=0}^{\infty}$, defined by (2) through (4) for any given K_{-1}, is convex.

iii) Write a matlab routine that, for given values of α, β, γ, δ, and ϕ, calculates the deterministic steady state values of *C*, *I*, and *K* under the assumption that $A_t \equiv 1$.

iv) Write a second matlab routine that, for given values of α, β, γ, δ, and ϕ, and of the steady state values of the variables, returns the matrices $\Gamma_0, \Gamma_1, C, \Pi,$ and Ψ of an equation system in the canonical form of the "Solving Linear Rational Expectations Models" notes that represents the Euler equations and constraints of this growth model, linearized around the deterministic steady state. Note that the only exogenous random variable in the model, as we have formulated it, is *A*, and we will assume that A_t is i.i.d. across time with mean 1. Here, because you are going to use the computer, it probably does not pay to do algebra in an attempt to reduce the dimension of the system. Assuming we look for a solution in which the positivity constraints (4) are not binding, you will have two Lagrange multipliers in addition to *C*, *I*, and *K*, and five equations – three Euler equations and two constraints.

v) Use your matlab routines to check existence and uniqueness and, over the range where there is a steady state and existence and uniqueness both hold, plot the largest root of the system

a) when a=.4, b=.95, f =.1, d=.9, as a function of g over the range .4 to 8, in increments of .4.

b) when $g = 2$ and other parameters (except f) are as in (a), as a function of f vary over the range (.05,1) in increments of .05.

Comment on the extent to which your results bear out or do not bear out the view that consumption is approximately a random walk.

II. Using `gensys.m`

The matlab routine `gensys.m` is available for downloading in the same directory that you get to when you click on "Solving Linear Rational Expectations Models" on the course reading list. You need also the auxiliary routines, or ".m" files in that same directory.

`gensys` takes as input the matrices $\Gamma_0, \Gamma_1, C, \Pi$, and Ψ for a standard linear first-order rational expectations system, plus an optional argument *div*, and gives as output the matrices *G1*, *C*, *impact*, *fmat*, *fwt*, *ywt* , *gev*, and *eu*. If *div* is not present in the argument list, the routine assumes that any root with absolute value greater than one is ruled out. Otherwise, it assumes that roots with absolute value greater than *div* are ruled out. The full list of output arguments is needed only to characterize a "forward" solution, in which exogenous disturbances are possibly serially correlated. When, as in this problem, the exogenous variables *z* are not serially correlated, the stable solution is of the form

$$y_t = G1 \cdot y_{t-1} + impact \cdot z_t \ . \tag{5}$$

The returned matrix *eu* has elements that are usually zero or 1. When the first element is 1, a solution exists, otherwise it does not. When the second element is 1, the solution is unique, otherwise it is not. Both elements can emerge as -2. This occurs when the equation system is less than full rank – usually some equation is just a linear combination of some others. it is possible to have both elements zero. This can occur because usually when a solution does not exist, it is possible to generate a "solution" by restricting the behavior of *z*, so that it lies in some less-than-full-rank space (and the program returns such a solution). Even then, though, it can turn out that this ad hoc pseudo-solution is not unique.

The returned matrix *gev* determines the generalized eigenvalues for the system. The ratio of the second column of *gev* to the first column is the set of eigenvalues of $\Gamma_0^{-1}\Gamma_1$ in the special case where Γ_0 is full rank. The generalized eigenvalues can sometimes help in diagnosing the source of problems when existence and uniqueness fail. The eigenvalues of the solved system are those of $G1$, and can be found either by the matlab command `eig(G1)`or by looking at the components of the second column of *gev* divided by its first column that are less than or equal to one (or *div*) in absolute value.