Lecture 7: New Keynesian FTPL, part 2

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Road map

We will go through setting up, solving, and roughly calibrating a New Keynesian model where we can compare active-money/passive-fiscal policy with an active-fiscal/passive-money specification.

The model shows that responses to monetary policy shocks in an AF/PM setting can look almost the same as they do in an AM/PF setting, even though the AF/PM setting implies large effects of fiscal shocks on inflation, while the AM/PF setting has no effect at all of fiscal shocks on inflation.

The model is based on Sims (2011). It differs in that it corrects an error in the FOC's (and hence in the displayed impulse responses) and in that it is expanded to allow AM/PF as well as AF/PM rules. The error in the FOC's was found by John Cochrane as he tried to reproduce the results in the paper.

What the model matches and misses

- Unlike the simplest constant-r, constant-τ AF/PM models, this one produces negative impacts of monetary contraction on output and prices and implies non-jumpy time paths for both consumption and prices.

- It still models primary surpluses as persistent, however, so primary deficits counterfactually lower the market value of debt immediately

Household problem

$$\max_{C,B,A} \int_0^\infty e^{-\beta t} \left(\frac{C^{1-\sigma}}{1-\sigma} - \tfrac{1}{2}\psi \dot{c}^2 \right) dt$$

subject to $$C + \frac{\dot{B}}{P} + \frac{\dot{A}}{aP} + \tau = Y + \frac{rB}{P} + \frac{A}{P}\,, \qquad B \geq 0\,, A \geq 0\,.$$

We will assume the government supplies no short debt B, so $B \equiv 0$ in equilibrium. It appears here to generate via FOC's an arbitrage condition between the short rate r and the long rate a. The CRRA utility has intertemporal elasticity σ, and its limit as $\sigma \to 1$ is log utility. C is the level of consumption and lower case c is $\log C$. Note that this is a Keynesian model, with demand-determined output and no investment or capital. So $Y = C$ and Y is not constant.

Phillips curve

We omit derivation of the Phillips curve from a price-setter's optimization problem and simply specify it as

$$\dot{\pi} = \mu\pi - \delta c$$

Note that, like the Euler equation FOC's we derive below, the Phillips curve is forward-looking, i.e. π (but not P itself) can jump.

GBC

The GBC is

$$\frac{\dot{A}}{aP} + \tau = \frac{A}{P} \,.$$

Note that a is the "long rate", the coupon rate of return on a consol at market value. It is also one over the price of the consol. In some previous slides we worked with $q = 1/a$ instead.

To make things look more familiar, we will define $b = \frac{A}{aP}$ and write the GBC as

$$\dot{b} + \tau = \left(a - \pi - \frac{\dot{a}}{a} \right) b$$

Fiscal and monetary policy

$$\dot{\tau} = \zeta \left(-\tau + \bar{\tau} + \omega \dot{c} + \chi \Big(b - \frac{\bar{\tau}}{\beta} \Big) \right) + \varepsilon_\tau$$

$$\dot{r} = \gamma(-r + \beta - \theta\pi - \phi\dot{c}) + \varepsilon_m$$

Our version of active fiscal policy will have $\chi = 0$, but ω, determining the "automatic stabilizer" component of fiscal policy, will be positive for both AF and PF.

Our version of AM will have $\theta > 1$, while our version of PM will have $0 < \theta < 1$.

Note $p = \log P$, $\pi = \dot{p}$.

FOC's

$$\partial C: \qquad e^{(1-\sigma c)} + \psi \cdot (\ddot{c} - \beta \dot{c}) = \lambda e^{c}$$

$$\partial A: \qquad \frac{-\dot{\lambda} + \beta + \pi + \dot{a}/a}{Pa} = \frac{\lambda}{P}$$

$$\partial B: \qquad \frac{-\dot{\lambda} + \beta + \pi}{P} = \frac{r\lambda}{P}$$

Simplified FOC's

$$r = a - \frac{\dot{a}}{a}$$

$$\rho = \frac{-\dot{\lambda}}{\lambda} + \beta$$

$$r = \rho + \pi$$

The ρ in the above equations is the real interest rate. We introduce it just to simplify notation.

Root counting

All three of the simplified FOC's are forward-looking, as is the Phillips Curve and the C FOC. But since as we have noted, ρ could be substituted out of the simplified FOC's, leaving only two equations, we only need 4 unstable roots. (The gensys package recognizes that there is a redundant forward-looking equation if fed the full system with 5 forward-looking equations.)

Solving it

This system is non-linear, but it is a system of ordinary differential equations. In principle numerical solution should be possible. One would specify initial conditions and specify that the system reaches its steady state at some distant date. There are sophisticated programs to solve such problems. However, they do not necessarily succeed with economic models like this one without a lot of hand tuning.

We will proceed with finding a local linear solution. That is, we replace the system of equations with linear equations describing the local behavior of the system in the neighborhood of its steady state. This allows solutions in which the iterative numerical calculations are buried in eigenvalue decomposition routines, exponentiation, etc. which are very stable without hand tuning.

Using `gensys`

- The algebra involved in finding the steady state of a system this size and differentiating the equations of the system at the steady state values of the variables is tedious and error-prone.
- The `gensys` R package and the DYNARE system (for Matlab) allow you to enter the system as ordinary R or matlab expressions, in their original non-linear form.
- The program then finds expressions for the derivatives needed for linearization, finds the steady state of the system, and solves the model.
- In `gensys` at least (and probably also DYNARE) auxiliary programs calculate impulse responses and plot them.

Example of equations formatted for entry into `gensys`

- The equations are listed with a name on one line, then the equation on the next. Forward looking equations are identified by having an asterisk at the end of their name.

- Time derivatives are indicated by having `dot` at the end of the variable names.

- The equations appear as expressions that evaluate to 0, so they contain no equal signs.

- After the equations come `vlist`, a list of variable names, `param`, a list of parameter names, and `shock`, a list of exogenous shock names.

Notations mapping between code and mathematical model

Though these equations describe the same model as we have laid out in mathematical form in the first part of these slides, the parameters and variable names don't correspond exactly. The `thet` and `phi` in the code correspond to $\theta\gamma$ and $\phi\gamma$. Inflation π corresponds to `inf`. β, the discount rate, corresponds to `rhobar`. The Phillips curve coefficients μ, δ correspond to `bet, delt`. The code also allows zero-mean stochastic shocks to τ, called `tau`, and a mean value of τ, `taubar`.

```
mpolicy
rdot - (-gam * (r - rhobar) + thet * pdot + phi * cdot + epsm)

Fisher*
r - (rho + pdot)

IS*
rho - (-lamdot/lam + rhobar)

gbc
bdot - (-b * pdot - b * adot/a + a * b - taubar - tau)

termstruc*
r - (a - adot/a)

phcurve*
```

```
infdot - (bet * pdot - delt * c - epspc)

infdef
pdot - inf

fpolicy
taudot -(zeta * (-tau + taubar + omega * cdot   + chi * (b - taubar / rhobar))

lamdef*
lam - (exp(-sig * c) + psi * (ccdot - rhobar * cdot) * exp(-c))

ccdef
cc - cdot

vlist
cc c lam tau inf p r rho b a

param
```

```
psi sig omega delt bet taubar rhobar gam phi thet chi zeta

shock
epsm epspc epstau
```

Interpreting the shocks

There are two ways to think of the exogenous shocks $\varepsilon_\tau, \varepsilon_m$ (or `epstau, epsm`).

1. You can think of the shocks as zero-mean, serially uncorrelated Gaussian "white noise" processes.

2. You can think of them as usually zero but at random moments causing unanticipated, zero-mean, jumps in the model's initial conditions. This latter interpretation makes them what are sometimes called "MIT shocks".

Representing the variables as functions of the history of shocks

Recall from lecture 5 that `gensys` delivers a solution in the form

$$Dy = Gy + c + H\varepsilon \,.$$

Actually in that lecture the $H\varepsilon$ term was missing, but I assured you that the solution procedure was the same whether or not a shock term was present. If y and G were scalars, since the solution is stable, G would be negative and we could solve the equation backward to get

$$y_t = \int_0^\infty e^{Gs} H\varepsilon_{t-s}\, ds + (1+G)^{-1}c \,.$$

With the appropriate definition of a "matrix exponential" e^G for matrix G, it turns out that this formula also works for vector y and matrix G. The matrix exponential is defined by

$$e^G = I + G + \frac{1}{2}G^2 + \cdots + \frac{1}{n!}G^n + \ldots$$

when G has a Jordan decomposition $G = P\Lambda P^{-1}$ with Λ diagonal and eigenvalues λ_i on the diagonal, we can also write

$$e^G = Pe^{\Lambda}P^{-1}\,,$$

where e^{Λ} is (clearly, from the definition of the matrix exponential above) a diagonal matrix with e^{λ_i} in the ith position on the diagonal.

So we can represent the vector y_t that is the solution to our model as

$$y_t = \int_0^\infty e^{Gs} H \varepsilon_{t-s} \, ds + (I - G)^{-1} c \, .$$

The only difference between this and the one-dimensional version is the replacement of "1" with an "I". The impulse responses of the system are the elements of the $e^{Gs}H$ matrix as functions of s. They can be thought of as describing how the system deviates from its initial path upon arrival of an "MIT shock", or as describing how the weights on past values of white noise shocks decay with lag length.

Monetary and fiscal policy shock irf's, AF/PM, then AM/PF

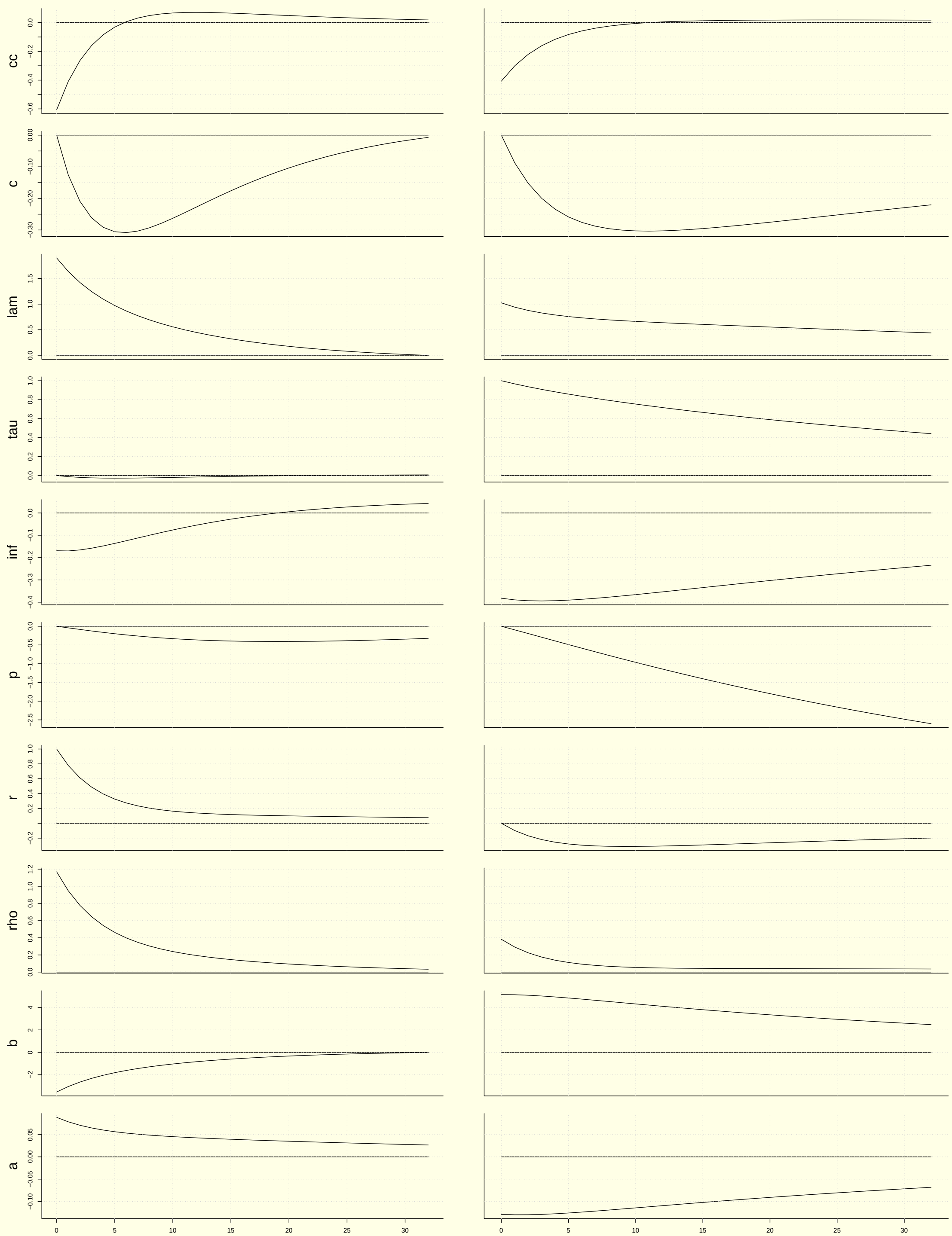
Though the irf plots appear on the next two slides, they may be easier to see if you open the original pdf files, which appear in the same directory as these slides on the course web site.

The parameter values used in calculating these irf's are, for the AF/PM specification,

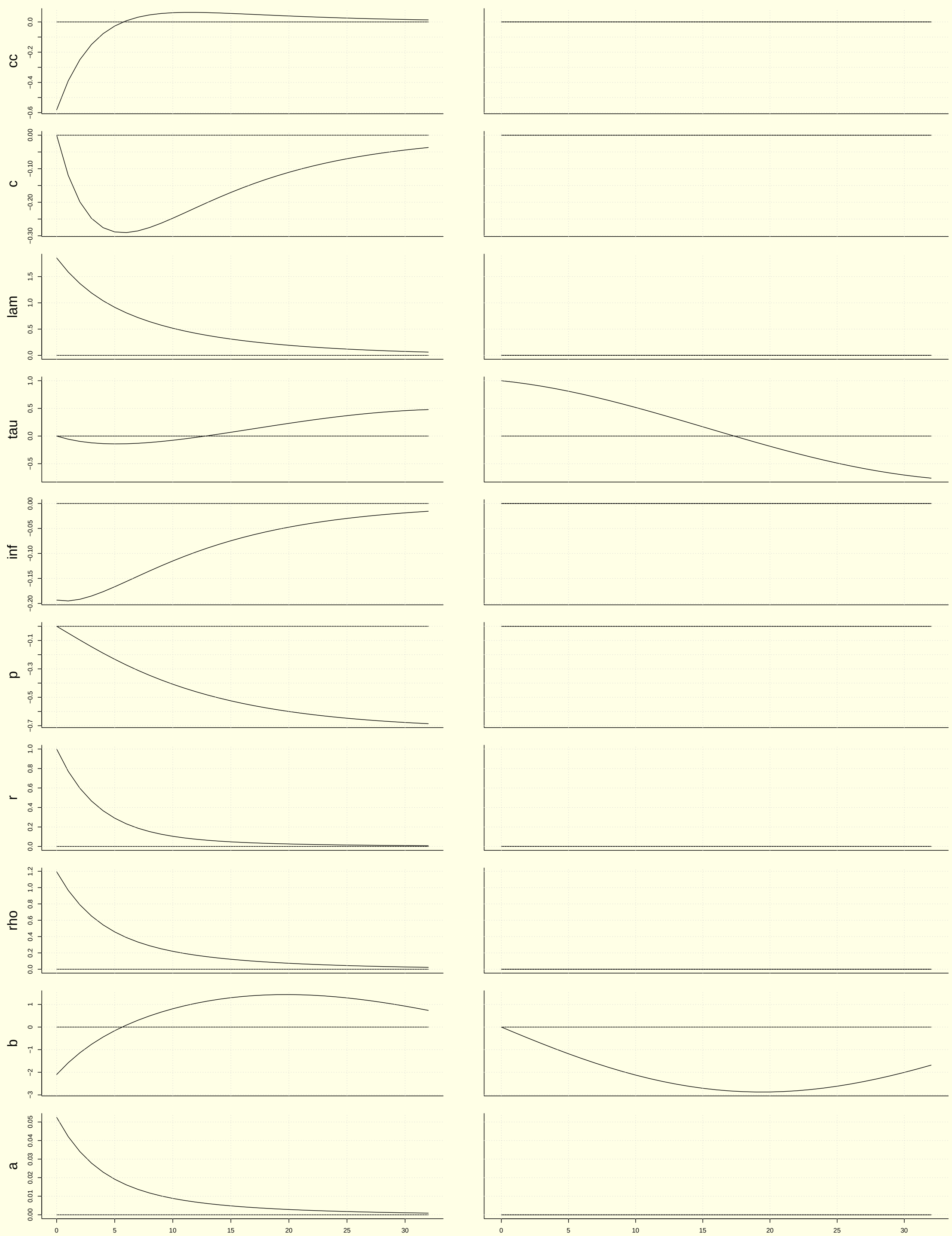
psi	sig	omega	delt	bet	taubar	rhobar	gam	phi	thet	chi	zeta
2.00	2.00	1.00	0.20	0.10	0.10	0.05	0.50	0.75	0.40	0.00	0.10

For the AM/PF specification the only changes are that `thet` rises to .6 and χ becomes 1.0 instead of 0. On each slide, the left column is the responses to the monetary policy shock ε_m and the right-hand column plots the responses to ε_f.

NKafpm irf's



NKampf irf's



Economic interpretation of the response to ε_m

- The first columns, the responses to the monetary policy shocks, show the effects of a one percentage point shock to r, generated by ε_m.

- In both models it raises the interest rate by 1 percent, lowers inflation by about .2 percent, and causes a smooth decline in consumption by abou .3 percent over a year or so. (The initial inflation responses are the same, but the vertical scales of the plots are different in the two models.)

- The long run responses to ε_m do differ: in particular the response of inflation eventually reverses sign, so that the contractionary policy shock has eventually increased, rather than decreased inflation.

Economic interpretation of the response to ε_τ

- While the monetary policy shock responses are similar in the two models, the responses to ε_{tau} are quite different.

- The contractionary fiscal policy shock, of about 10 percent of the mean level of τ, has no effect at all on inflation in the AM/PF version of the model, whereas it has a large effect on inflation in the AF/PM version.

- And we see the problem arising from the persistence of τ: b drops in response to the τ shock in the AM/PF version, but it rises, for the reasons we have discussed, in response to the τ shock in the AF/PM version.

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References

SIMS, C. A. (2011): "Stepping on a Rake: The Role of Fiscal Policy in the Inflation of the 1970's," *European Economic Review*, 55, 48–56.