INTERNATIONAL MACROECONOMIC LINKAGES

1. A CATALOG OF LINKAGES

Capital flows: They should equate MPK across the world, and they should make U'''s move in proportion worldwide. (Technical efficiency and risk sharing.)

Import and export supply and demand: This is the traditional Keynesian link. E.g., expansion abroad raises demand for exports, leads to expansion at home.

Common technology shocks: Economic theory does not say much about this, but if "technology" is something like scientific knowledge (which it might not be), it should spread worldwide with little delay.

2. CAPITAL FLOW PARADOXES

Feldstein-Horioka: The "puzzle": Investment and savings are strongly correlated across countries. Why is this a puzzle? It depends on an informal model, in which technology generates differing investment opportunities across the world and the pool of world savings is allocated efficiently across them. Income and tastes would also generate a varying pattern of savings changes across countries, but there seems no obvious reason why these should be linked to the technology shocks.

Home bias: It is a fact that investors tend to hold "local" assets. On the face of it, this seems inconsistent with CAPM reasoning about portfolio allocation. One would think local assets would have returns positively correlated with holders' untraded risk.

: These facts would be easy to explain in a model where people were forbidden to own assets in other countries or to borrow and lend internationally. But the facts show up in the most advanced economies, where it is apparently easy to purchase foreign assets and where expected return differentials across countries are small.

3. MACROECONOMIC CURRENT ACCOUNT MODELS

These are models in which trade plays no role. There is a single commodity, and the current account represents international borrowing, lending, and investment. They help us understand the baseline — what's a "paradox" and what isn't.

Model 0: Expected return equalization vs. risk sharing: Countries are $i = 1, 2$. Agents in i solve

$$\max_{C_i(s), Bi(s)} E\left[\sum_{t=0}^{\infty} \beta^t U(C_i(t))\right]$$

s.t.

$$C_i(t) + B_i(t) = \rho_{t-1} B_i(t-1) + Y_i(t)$$
$$B_i(t) \geq -\bar{B}$$

Market clearing: $\qquad B_1(t) + B_2(t) = 0$

SRC: $\qquad C_1(t) + C_2(t) = Y_1(t) + Y_2(t)$

Planner's problem: Ignore the need to use borrowing and lending to smooth consumption. Just maximize a weighted sum of utilities subject to the SRC above:

$$\max_{C_i(s)} E\left[\sum_{t=0}^{\infty} \beta^t w U_1(C_1(t)) + (1-w) U_2(C_2(t))\right]$$

This problem is non-dynamic. No resources are movable across time. So it can be solved period-by-period and implies simply $wU'(C_1(t)) = (1-w)U'(C_2(t))$, all t. If $w = .5$, $C_1 \equiv C_2$.

Competitive equilibrium with borrowing and lending: This problem is hard to solve. Even with convenient forms for U and for the Y stochastic process, the need for a no-Ponzi condition creates complications. We can use local linearization around a $B_i = 0$ deterministic steady state, however.

FOC's

∂C_i: $\qquad\qquad\qquad U_i'(C_i(t)) = \lambda_i(t)$

∂B_i: $\qquad\qquad\qquad \lambda_i(t) = \beta \rho_t E_t [\lambda_i(t+1)]$

The nature of the solution: C is $\rho - 1$ times the discounted present value of future Y plus current wealth $\rho_{t-1} B(t-1) + Y(t)$. Thus most of "transient income" is saved. But some isn't. Y shocks affect permanent income. They are smoothed out, but they have persistent effects via their effects on wealth. ρ fluctuates with the ratio of *aggregate Y* to its normal value, with ρ high when aggregate expected ΔY is temporarily high. But individual C's and B's behave like martingales with opposite-signed shocks. This implies that the quality of the local linear approximation eventually gets very bad. This equilibrium is said to show **consumption smoothing** via asset markets, without **risk sharing**.

Implementing complete markets: The usual rule: one asset per independent source of uncertainty, plus a risk-free asset, to implement complete markets, unless the assets are chosen "just right". In discrete time, with continuously

distributed disturbances, one needs in general infinitely many assets with even one shock. A "single source of disturbance" has to be read as a single 0-1 random variable. A normally distributed disturbance has to be thought of as made up of infinitely many little 0-1 variables, and hence requires infinitely many assets for a complete market. Here, because of all the symmetry, it is enough to introduce two assets, one paying Y_1 as a dividend, the other paying Y_2 as a dividend. The new constraints are:

$$
\begin{aligned}
C_1(t) - Q_1(t)S_1(t) + Q_2(t)S_2(t) = \\
- (Q_1(t) + Y_1(t))S_1(t-1) \\
+ (Q_2(t) + Y_2(t))S_2(t-1) + Y_1(t) \\
C_2(t) + Q_1(t)S_1(t) - Q_2(t)S_2(t) = \\
+ (Q_1(t) + Y_1(t))S_1(t-1) \\
- (Q_2(t) + Y_2(t))S_2(t-1) + Y_2(t)
\end{aligned}
$$

Each agent also must face a no-Ponzi condition, e.g. $S_1(t) \leq 1$, $S_2(t) \geq 0$ for agent 1. This specifies that, having no Y_2 endowment, agent 1 can't sell stock in it, and having an endowment stream of Y_1, the agent can't sell rights to more than that entire endowment stream. These restrictions are not as natural as they sound. In a model where agent 1 were more risk averse than agent 2, it might be required for complete markets equilibrium that agent 1 sell the S_2 asset, for example.

4. CHECKING EQUILIBRIUM CONDITIONS

Competitive equilibrium with these assets can implement the $C_1 \equiv C_2 \equiv (Y_1 + Y_2)/2$ allocation. This emerges when $S_1 = S_2 = .5$, and it can be verified that there is a set of prices (the same for both assets, for i.i.d. Y) that makes competitive agents satisfied with this allocation. There is *no* international asset trade in this equilibrium. Consumption is equalized because everyone owns the same portfolio (including endowments) and thus has the same income.

Country 1 FOC's:

Country 1

∂C_1: $\qquad U'(C_1(t)) = \lambda_1(t)$

∂S_1: $\qquad Q_1(t)\lambda_1(t) =$

$\qquad\qquad \beta E_t\left[\left(Q_1(t+1) + Y_1(t+1)\right)\lambda_1(t+1)\right]$

∂S_2: $\qquad Q_2(t)\lambda_1(t) =$

$\qquad\qquad \beta E_t\left[\left(Q_2(t+1) + Y_2(t+1)\right)\lambda_1(t+1)\right]$

TVC: $\qquad \lim_{t\to\infty} E\left[\beta^t Q_1(t)\lambda_1(t)S_1(t) - Q_2(t)\lambda_1(t)S_2(t)\right] = 0$

The postulated equilibrium, with

$$C_1(t) \equiv C_2(t) \equiv \bar{Y}(t) = \frac{Y_1(t) + Y_2(t)}{2},$$

$$S_1(t) \equiv S_2(t) = .5$$

$$Q_1(t) \equiv Q_2(t)$$

certainly satisfies the budget constraints, and it is easily seen that it implies that for each agent endowment plus dividend income is the same, i.e. $\bar{Y}$, so that the agent follows the proposed allocation by simply consuming income. The question that remains is whether we can choose Q_i so that the agents' FOC's are satisfied. There is a solution in which the price for both stocks, $\bar{Q}$, is a function of $\bar{Y}(t)$ alone. This $\bar{Q}$ function has to satisfy, e.g.,

$$\bar{Q}(t)U'(\bar{Y}(t)) =$$

$$\beta E_t\left[U'(\bar{Y}(t+1))\left(\bar{Q}(\bar{Y}(t+1)) + Y_1(t+1)\right)\right]$$

This may look like a mess, but with i.i.d. Y_i, the right hand side is a constant, because it depends only on random variables that are i.i.d. across time and therefore has conditional expectation E_t equal to its unconditional expectation. The FOC's for agents $i = 1, 2$ w.r.t. assets $j = 1, 2$ all have the same form, so the same $\bar{Q}(t)$ solves them all. The actual function $\bar{Q}(\cdot)$ depends on the distribution of Y_i and on the form of U.