EXERCISE ON RANDOM LAGRANGE MULTIPLIERS AND TVC

(1) Suppose in the LQPY model as discussed in class we replace the standard quadratic utility function $U(C_t) = C_t - \frac{1}{2}C_t^2$ with $U(C_t) = C_t - \frac{1}{2}C_t^2 - \frac{1}{4}(C_t - C_{t-1})^2$. This makes the consumer dislike rapid changes in C.
  (a) Show that the objective function in this modified model is concave.
  (b) Using the $C_t \leq W_t$ version of the borrowing constraint, and this modified objective function, find the Euler equations and transversality conditions.

(2) Consider the growth model without discounting:

$$\max \sum_{t=0}^{\infty} \log(c_t/\bar{c}) \tag{1}$$

subject to

$$c_t + k_t \leq k_{t-1}^{\alpha} \,. \tag{2}$$

We pick $\bar{c}$ to be $(1-\alpha)\alpha^{\alpha/(1-\alpha)}$, which we hope will be the stead-state value of c (so that the undiscounted objective function will converge).
  (a) Find the Euler equations and transversality condition for this problem.
  (b) Verify or refute the conjecture that if there is any solution to the problem that implies $c_t \to \bar{c}$, it fails to satisfy the standard transversality condition.
  (c) There is an optimal solution that makes c converges to $\bar{c}$. Is this is an example of a model in which the TVC is not a necessary condition for an optimal solution, or does our general version of the TVC apply here? Does the model violate assumptions of our proof of the sufficiency for the FOC's with the TVC?