EXERCISE ON THE PRICE LEVEL AND OG NATURAL RESOURCES

(1) Consider the model with money and debt that we discussed in class, where agents maximize

$$E\left[\sum_{t=0}^{\infty}\beta^t \log C_t\right] \text{ subject to} \tag{1}$$

$$C_t\cdot(1+\gamma f(v_t)) + \frac{B_t+P_t}{P_t} + \tau_t = \frac{R_{t-1}B_{t-1}+M_{t-1}}{P_t} + Y_t \tag{2}$$

$$v_t = \frac{P_tC_t}{M_t}\,. \tag{3}$$

Their choice variables are C, B and M.

The government has the budget constraint

$$\frac{B_t+M_t}{P_t} + \tau_t = \frac{R_{t-1}B_{t-1}+M_{t-1}}{P_t}\,. \tag{4}$$

We will assume it has a passive fiscal policy and a fixed-M monetary policy, i.e.

$$\tau_t = -\phi_0 + \phi_1\frac{B_t}{P_t}\,,\quad \phi_1 > \beta^{-1}\,,\quad \phi_0 > 0 \tag{5}$$

$$M_t \equiv \bar{M}\,. \tag{6}$$

In class and in the assigned paper that discusses this model we considered conditions for existence and uniqueness of equilibrium with $f(v) = v$ and with $f(v) = v/(1+v)$. In this exercise you are to check conditions for existence and uniqueness of equilibrium with $f(v) = v^q$ and with $f(v) = v^q/(1+v^q)$. Determine what range of values for γ and q imply existence and uniqueness. Explain the reasoning behind your answers. You can assume that q and γ are positive.

(Question 2 on next page)

(2) This problem considers an OG model with an exhaustible resource. Each generation works when young, using its earnings to finance consumption and purchase of resource stocks from the old. Consumption goods can be produced only by using up some of the resource.

The agents' utility function, for those born at t, is $\log(C_1(t)C_2(t+1))$. Their constraints are

$$C_1(t) + P_t R_t = (R_t - R_{t+1})^\alpha \tag{7}$$

$$C_2(t+1) = P_{t+1} R_{t+1} \tag{8}$$

$$R_t \geq R_{t+1} \geq 0\,. \tag{9}$$

Agents treat P_t as given and maximize over $C_1(t)$, $C_2(t+1)$, $R(t)$, and $R(t+1)$. We are imposing market clearing implicitly by using the same symbol for the resources $R(t)$ sold by the old at t and for the resources bought by the young at that date. We assume $0 < \alpha \leq 1$ and constant population.

(a) Find a competitive equilibrium for the model. Is it unique?

(b) Consider two possible tax schemes, aimed at preventing the welfare of successive generations decline as fast as in the original competitive equilibrium. In tax scheme 1, the young pay a tax as a fixed proportion of the value of $P_t R_t$, with the proceeds distributed as a lump-sum transfer to the old. In scheme 2, they pay a tax as a fixed proportion of $P_t \cdot (R_t - R_{t+1})$, their resource usage, again with the proceeds transferred lump-sum to the old. Do both tax schemes improve the relative welfare of future generations? Does either scheme have an efficiency advantage over the other?