LECTURE 5: STOCHASTIC TAXES ON NOMINAL BONDS

1. THE BARRO MODEL

We discuss Barro's model, generalized to allow non-quadratic deadweight loss.

- Conclusion: it is optimal not to plan to pay down existing public debt; both taxes and debt are expected to stay constant under optimal policy.
- Drastic simplifying assumptions, making mathematics simple, indeed equivalent to the structure of the permanent income model.
- Assumes deadweight loss from constant revenues is constant, i.e. something like a labor tax, not a capital tax.

2. GOVERNMENT'S PROBLEM

$$\text{Obj.fcn:} \qquad \max_{\tau_s, B_s} E\left[\sum_{t=0}^{\infty} -\ell(\tau_t)\beta^t\right] \tag{1}$$

$$\lambda: \qquad B_t + \tau_t \geq \rho_{t-1}B_{t-1} + G_t \tag{2}$$

$$\text{Fisher:} \qquad \rho_t = \beta^{-1} \tag{3}$$

$$\text{no Ponzi:} \qquad E[\beta^t B_t] \underset{t\to\infty}{\longrightarrow} = 0 \tag{4}$$

- (3) implies absence of risk aversion, or else that consumption is constant.
- By using a function of τ alone to stand for deadweight loss from taxes and τ to stand for total revenues, the model avoids treating explicitly fluctuations in the tax base and possible effects of tax rates on the tax base.

3. FOC's

$$\partial\tau: \qquad \ell'(\tau_t) = \lambda_t$$

$$\partial B: \qquad \lambda_t = E_t\lambda_{t+1}$$

- The FOC's deliver immediately the conclusion that $\ell'(\tau_t)$ is a martingale.
- If ℓ' is everywhere positive, this implies that in the absence of uncertainty, τ itself will be set at a constant.
- The government's no-Ponzi constraint (4) together with its budget constraint require that, when τ_t itself is a martingale, the planned constant level of taxes match the interest rate times current debt plus the discounted present value of future G:

$$\tau_t = (\beta^{-1} - 1)\left(B_t + E_t\left[\sum_{s=1}^{\infty}\beta^s G_{t+s}\right]\right) \tag{5}$$

4. UNCERTAINTY

- If ℓ is quadratic, then ℓ' is linear and τ itself is implied to be a martingale.
- It is natural to suppose that ℓ must, at least for high values of τ, increase more rapidly than a quadratic, since tax rates and current output are inherently bounded, and one supposes that as tax revenues approach their maximum, the deadweight loss must increase very rapidly.
- So assume that ℓ' is convex, and thus that τ is a concave function of ℓ'.
- $Ex = \bar{x} \Rightarrow Ef(x) \leq f(\bar{x})$ for any concave function f, so $E\tau_{t+1} \leq \tau_t$.
- That is, in the presence of uncertainty expected revenues decline over time. In effect, uncertainty makes it optimal to tax more heavily now as insurance against being driven to very inefficient high tax rates in the future.
- Even in the quadratic case considered by Barro, though expected future τ is constant, actual τ adjusts, period-by-period, to any random disturbances in current and future G. It is not hard to verify that under these conditions B, like τ, is a martingale.

5. ADDING A PRICE LEVEL TO THE BARRO MODEL

$$\lambda: \qquad \frac{B_t}{P_t} + \tau_t \geq \rho_{t-1}\frac{B_{t-1}}{P_t} + G_t \tag{6}$$

$$\mu: \qquad 1 \leq \beta\rho_t E_t\left[\frac{P_t}{P_{t+1}}\right] \tag{7}$$

$$\lim_{t\to\infty} E\left[\beta^t \frac{B_t}{P_t}\right] = 0 \text{ or } B_t \geq 0 \tag{8}$$

$$\partial\tau: \qquad \ell'(\tau_t) = \lambda_t$$

$$\partial B: \qquad \lambda_t = \beta\rho_t E_t\left[\lambda_{t+1}\frac{P_t}{P_{t+1}}\right]$$

$$\partial\rho: \qquad \beta B_t E_t\left[\frac{\lambda_{t+1}}{P_{t+1}}\right] = \mu_t\beta E_t\left[\frac{P_t}{P_{t+1}}\right]$$

$$\partial P: \qquad \frac{\lambda_t}{P_t^2}\left(B_t - \rho_{t-1}B_{t-1}\right) = \mu_t\rho_t\beta E_t\left[\frac{1}{P_{t+1}}\right] - \mu_{t-1}\rho_{t-1}\frac{P_{t-1}}{P_t^2}$$

6. ALGEBRA

From the B and ρ FOC's, together with the private FOC (7), we can derive a relation of μ_t to λ_t:

$$\mu_t = \frac{B_t}{P_t}\lambda_t \tag{9}$$

Using this and (7) again in the P FOC, we get

$$\lambda_t(B_t - \rho_{t-1}B_{t-1}) = \lambda_t B_t - \lambda_{t-1}B_{t-1}\rho_{t-1}\,,$$

which reduces to $\lambda_t = \lambda_{t-1}$, and thus via the τ FOC to $\tau_t = \tau_{t-1}$.

7. DISCUSSION

The conclusion we have derived here applies only *after* $t = 0$. This is because it uses, in time-t FOC's, expectational constraints dated $t-1$. In particular, the FOC w.r.t. P_0 is not the P FOC given above with t set to zero, but instead

$$\partial P_0: \qquad \frac{\lambda_0}{P_0^2}\left(B_0 - \rho_{-1}B_{-1}\right) = \mu_0\rho_0\beta E_0\left[\frac{1}{P_1}\right].$$

which reduces to $\lambda_0\rho_{-1}B_{-1}/P_0^2 = 0$. We postpone momentarily discussing the implications of this initial-date condition. Also, we have assumed an interior solution. If there is a $B \geq 0$ no-Ponzi condition, and we were up against this constraint last period, the $\tau_t = \tau_{t-1}$ conclusion no longer applies. We postpone discussion of this temporarily also.

The intertemporal budget constraint of the government, which applies because of the no-Ponzi condition, requires that τ_0 be set to satisfy the same condition (5) as before, now interpreted as holding in expectation instead of exactly, and with B/P replacing B. How then does the government maintain fiscal balance, with τ_t constant but future G's changing stochastically? It need "do" nothing. The price level will adjust automatically to maintain the fiscal intertemporal budget constraint. A surprise increase in expected future G's leads to a surprise increase in P just sufficient to reduce B/P by enough to maintain intertemporal budget balance.

8. THE FIRST PERIOD

At $t = 0$, optimal policy satisfies the P_0 FOC above. For this to hold, with B_{t-1} and ρ_{t-1} non-zero, requires $\lambda_0 = 0$ or $P_0 = \infty$. $\lambda_t = 0$ is possible only at $\tau = 0$, where the marginal deadweight loss from taxation is zero. That in turn is possible only if debt is zero and current G is zero. If debt is issued at $t = 0$, then at $t = 1$ we will have $\tau_1 = \tau_0 = 0$, and this condition persists as long as $B_t > 0$. But with zero taxes and $G > 0$, there is no way that B can decrease, so this policy is unsustainable. If $\tau_0 > 0$, then the P_0 FOC requires $P_0 = \infty$, which is not technically possible. If P is ever infinite, then the future rates of inflation that enter into the model's equation are all undefined. However, it is true that welfare is higher the higher the initial P, so that optimal policy under the more realistic $B \geq 0$ constraint is in fact a very large initial surprise inflation that all but wipes out the real value of B_{-1}. Under the limit form of the no-Ponzi constraint (8), optimal policy is to switch the sign of B, converting outstanding government debt to government assets, in which holders make payments to the government instead of vice versa. The optimal initial P is then chosen so as to make earnings from the new government assets sufficient to cover the discounted present value of future G.

9. SUPPOSE $B_t = 0$

It can be optimal to cover G_t plus any outstanding debt entirely with τ_t, leaving $B_t = 0$, if there is a $B_t \geq 0$ constraint. However, it is easy to understand that this cannot be optimal if the expected ℓ'_{t+1} is well above the current ℓ'_t. If current taxes are lower than normal, positive borrowing only makes future taxes higher relative to current taxes, which is inefficient. In particular, in a period when G is unusually high, it will always be optimal to do some borrowing. From then on, it will be optimal to keep τ constant, unless it is not feasible to do so. In this latter case it will be optimal to inflate away essentially all the real value of the debt again, then set τ_t to a new, higher level, that will again be maintained constant until the next dire fiscal emergency that cannot be covered with an unexpected inflation tax on debt-holders.

Making these assertions rigorous requires specifying the stochastic properties of G_t. More detailed discussion of these "debt repudiation dynamics" are in my paper "Fiscal consequences for Mexico of adopting the dollar", on my website and on the course reading list.

10. Remarks about realism

Barro's model and its extension to consider inflation are both drastic simplifications, meant to illustrate a mechanism that should be taken seriously. Taxes should generally be smoothed, to some extent. Unanticipated inflation (and, in a model with long-term debt, unanticipated interest rate changes) are available as tools to smooth taxes. In reality unanticipated inflation has costs that are not incorporated in these models, because of price stickiness, money illusion, contracts written without inflation contingencies, etc. These have to be balanced off against the benefits of using unanticipated inflation to smooth taxes.

It also might be objected that the model's assumption that the government can announce future policies and be believed — i.e. can "commit"— is unrealistic. Note, however, that a government that really cannot commit can never issue any debt, because every period a government starting anew will want to repudiate debt issued by previous governments. Any interesting model of intertemporal fiscal policy must therefore assume some type of commitment ability on the part of the government. In fact, standard models of monetary policy without commitment can be regarded as internally contradictory, as they nearly always assume in the background a treasury that can issue debt.