EXERCISE ON DYNAMIC PANELS

We assume

$$\underset{2\times 1}{z_{it}} = \begin{bmatrix} y_{it} \\ x_{it} \end{bmatrix} , \tag{1}$$

$$z_{it} = c_i + A z_{i,t-1} + \varepsilon_{it} , \tag{2}$$

$$\varepsilon_{it} \mid \{z_{i,t-s}, s > 0\} \sim N(0, \Sigma) , \tag{3}$$

$$t = 1, \dots, T , \quad i = 1, \dots, M . \tag{4}$$

All the eigenvalues of A are assumed less than one in absolute value, and the z_{it} process for each i is assumed stationary over all t, not just $t = 1, \dots, T$ The ε_{it} processes are assumed independent across i.

(1) Explain in enough detail so that an undergrad RA who knows programming and matrix algebra, but not statistics or time series theory, could implement a joint pdf for $\{z_{it,t=1,\dots,T,i=1,\dots,M}\}$ conditional on $\{c_i, i = 1, \dots, M\}$, A, and Σ. Will maximizing this pdf over $\{c_i, i = 1, \dots, M, A, \Sigma\}$ produce consistent estimates of any of the parameters as $M \to \infty$ with T fixed? Note that because c_i is being treated as a "parameter" here and not given a distribution, your answer may depend on some assumption about the $\{c_i\}$ sequence.

(2) Now assume that we specify $c_i \sim N(\gamma, \Omega)$ for each i, and that c_i is independent across i. Show how to combine this assumed pdf for c_i with the conditional pdf you described in 1 to obtain a joint pdf for $\{z_{it}\}$ and $\{c_i\}$ conditional on γ, Ω, A, and Σ.

(3) Explain either
   (a) how to integrate $\{c_i\}$ out of the likelihood function you specified in 2; or
   (b) a MCMC algorithm to sample from the joint posterior on all the parameters, including the c_i, (which would deliver of course also a sample from the marginal posterior on the non-c_i parameters).

(4) Prove that use of this likelihood, assuming the model is correct, produces consistent estimates of the parameters as $M \to \infty$ with T fixed. (Note that the model implies that the z data for each group is a normally distributed vector with a distribution that is a known function of the model parameters.)

(5) We know that in a single equation model of the form

$$y_{it} = c_i + \rho y_{i,t-1} + \xi_{it}$$

the equation coefficients other than c_i can be estimated by the Arellano-Bond strategy of differencing the equation to eliminate c_i, then using lagged Δy or lagged y as instruments. However, as ρ approaches 1, these instruments become arbitrarily weak. Suppose we consider estimating the first equation of the system (2) above. Can we difference the equation and use lagged Δz or lagged z as instruments? Is there a condition analogous to $\rho \to 1$ in the single-equation case that would imply these instruments become arbitrarily weak?

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