BUBBLES AND FUNDAMENTALS

1. Discounting returns

- A promise to deliver money "later" is generally worth less than delivery of money now, even if the promise is completely credible.
- Such promises can be added and subtracted, so a promise to deliver $100 in two weeks and a promise to deliver $200 in two weeks should have prices that add up to the same as that of a promise to deliver $300 in two weeks.
- Furthermore, a promise to deliver, one week from now, exactly enough money to buy then a promise to deliver $100 one week further on, should have a current price exactly equal to the price of $100 delivered two weeks from now.

2. The market discount factor

- It turns out that these conditions, plus the assumption that any promise of this kind can be bought and sold freely, are enough to deliver the conclusion that there is a "market discount factor" R_t. This discount factor has the properties 1) that R_t can be known at time t and 2) that a promise to deliver the random return y_{t+1} at time $t+1$ has the value at time t

$$P_t = E_t\left[\frac{y_{t+1}R_{t+1}}{R_t}\right] .$$

- If there is no uncertainty, or if participants in the asset market have no risk aversion, R_t will be non-random. As an extreme simplification, we sometimes assume a constant interest rate, so that $R_t = (1+r)^{-t}$ and the pricing relation becomes simply $y_{t+1} = (1+r)P_t$.

3

- If there is uncertainty, but the return y_{t+1} is not uncertain (so we are talking about something like a treasury bill), then the return is the interest rate, and we have

$$\begin{aligned} y_{t+1} &= (1+r)P_t = (1+r)y_{t+1}E_t[R_{t+1}]/R_t \quad \text{i.e.} \\ 1/(1+r) &= E_t[R_{t+1}/R_t] \end{aligned}$$

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4. Pricing a random-yield asset

- This asset pays the random yield y_t each period.
- It is a stylized "stock" or "Lucas tree".
- Its price Q_t must satisfy

$$Q_t = E_t \left[\frac{R_{t+1}(y_{t+1} + Q_{t+1}))}{R_t} \right]$$